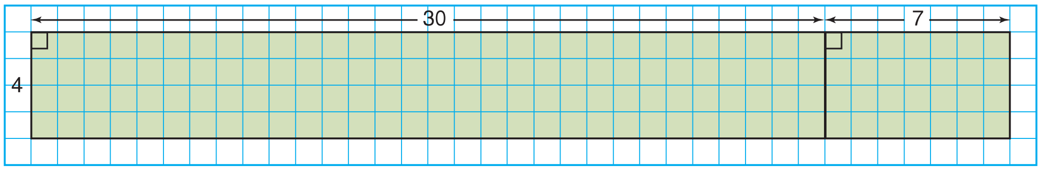
6.4 The Distributive Property

Focus Relate the distributive property to algebra.

How does this diagram model the product of 4×37?
What is the product?



Investigate

Work with a partner.

Gina's Trattoria Ristorante has only round tables.
Each table seats 5 people.
Some tables are in the lounge and some are on the patio.

➤ There are 20 tables in the lounge and 8 tables on the patio.
Draw a diagram to show how to calculate the total number of people who can be seated.

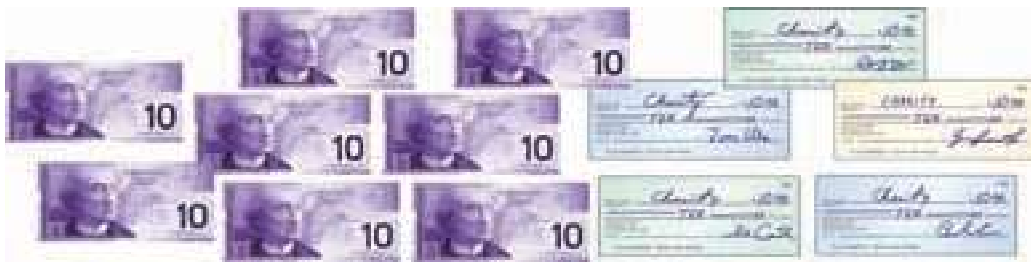
➤ Gina removes some tables from the lounge.
Use a variable to represent the number of tables that remain.
Write an algebraic expression for the number of people the restaurant can now seat.

Reflect & Share

Compare your diagram and expression with those of another pair of classmates.
If your diagrams and expressions are different, are both of them correct? How can you check?
If your expressions are the same, work together to write a different expression for the number of people the restaurant can now seat.

Connect

➤ A charity sells pots of flowers for \$10 each to raise money.
8 people pay with a \$10 bill.
5 people pay with a \$10 cheque.
The total amount of money collected, in dollars, is:



- Add the total number of bills and cheques, then multiply by 10:
$10 \times (8 + 5) = 130$

Or

- Multiply the number of bills by 10 and the number of cheques by 10, then add:
$10 \times 8 + 10 \times 5 = 130$

We write: $10 \times (8 + 5) = 10 \times 8 + 10 \times 5$

We can use the same strategy to write two equivalent expressions for any numbers of bills and cheques.

➤ Suppose you are selling event tickets for \$20 each.
Some people pay with a \$20 bill and some pay with a \$20 cheque.
Suppose b is the number of \$20 bills you receive,
and c is the number of \$20 cheques you receive.
The total amount of money you collect, in dollars, is:

- Add the total number of bills and cheques, then multiply by 20:
$20 \times (b + c)$

Or

- Multiply the number of bills by 20 and the number of cheques by 20, then add:
$20b + 20c$

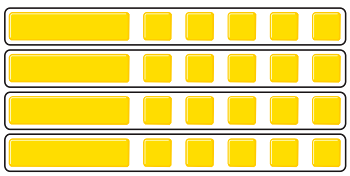
We write: $20(b + c) = 20b + 20c$

Multiplication has been *distributed* over addition.

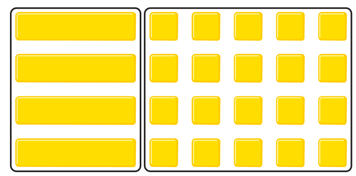
➤ We can also model the distributive property with algebra tiles.

For example:

To model $4(x + 5)$, you need 4 groups of 1 positive variable tile and 5 positive unit tiles.

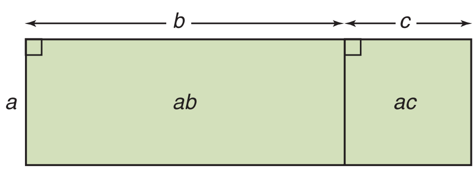


To model $4x + 20$, you need the same tiles, but their arrangement is grouped differently.



We can see that $4(x + 5) = 4x + 20$ because the two diagrams show the same numbers of tiles.

When an expression for the distributive property uses only variables, we can illustrate the property with this diagram.



$a(b + c) = ab + ac$

That is, the product $a(b + c)$ is equal to the sum $ab + ac$.

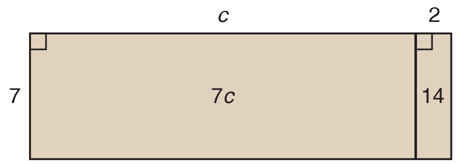
Example 1

Use the distributive property to write each expression as a sum of terms.
Sketch a diagram in each case.

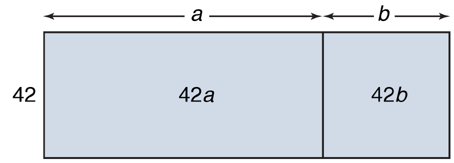
a) $7(c + 2)$ **b)** $42(a + b)$

A Solution

a) $7(c + 2) = 7(c) + 7(2)$
$= 7c + 14$



b) $42(a + b) = 42(a) + 42(b)$
$= 42a + 42b$



In *Example 1*, when we use the distributive property, we **expand**.

We can also use the distributive property with any integers.

Example 2

Expand.

a) $-3(x + 5)$

b) $-4(-5 + a)$

A Solution

a) $-3(x + 5) = -3(x) + (-3)(5)$
$= -3x - 15$

b) $-4(-5 + a) = -4(-5) + (-4)(a)$
$= 20 - 4a$

Subtraction can be thought of as "adding the opposite."
For example, $3 - 4 = 3 + (-4)$

Example 3

Expand.

a) $6(x - 3)$

b) $5(8 - c)$

A Solution

Rewrite each expression using addition.

a) $6(x - 3) = 6[x + (-3)]$
$= 6(x) + 6(-3)$
$= 6x - 18$

b) $5(8 - c) = 5[8 + (-c)]$
$= 5(8) + 5(-c)$
$= 40 - 5c$

Discuss the ideas

1. Look at the algebra tile diagrams on page 340.
   Why are the tiles grouped in different ways?
2. Can you draw a diagram to model each product in *Example 2*?
   Justify your answer.
3. Do you think the distributive property can be applied when there is a sum of 3 terms, such as $2(a + b + c)$?
   Draw a diagram to illustrate your answer.

Practice

Check

4. Evaluate each pair of expressions.
   What do you notice?
   a) i) $7(3 + 8)$
      ii) $7 \times 3 + 7 \times 8$
   b) i) $5(7 - 2)$
      ii) $5 \times 7 + 5 \times (-2)$
   c) i) $-2(9 - 4)$
      ii) $(-2) \times 9 + (-2) \times (-4)$

5. Use algebra tiles to show that $5(x + 2)$ and $5x + 10$ are equivalent.
   Draw a diagram to record your work.
   Explain your diagram in words.

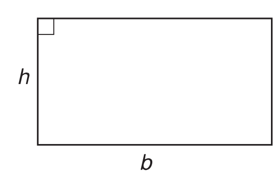
6. Draw a rectangle to show that $7(4 + s)$ and $28 + 7s$ are equivalent.
   Explain your diagram in words.

7. Expand.
   a) $2(x + 10)$ b) $5(a + 1)$
   c) $10(f + 2)$ d) $6(12 + g)$
   e) $8(8 + y)$ f) $5(s + 6)$
   g) $3(9 + p)$ h) $4(11 + r)$
   i) $7(g + 15)$ j) $9(7 + h)$

Apply

8. Expand.
   a) $3(x - 7)$ b) $4(a - 3)$
   c) $9(h - 5)$ d) $7(8 - f)$
   e) $5(1 - s)$ f) $6(p - 2)$
   g) $8(11 - t)$ h) $2(15 - v)$
   i) $10(b - 8)$ j) $11(c - 4)$

9. Write two formulas for the perimeter, P, of a rectangle.
   Explain how the formulas illustrate the distributive property.



10. Explain how you know $hb = bh$.
    Use an example to justify your answer.

11. Which expression is equal to $9(6 - t)$?
    How do you know?
    a) $54 - 9t$
    b) $96 - 9t$
    c) $54 - t$

12. Expand.

a) $-6(c + 4)$ b) $-8(a - 5)$

c) $10(f - 7)$ d) $3(-8 - g)$

e) $-8(8 - y)$ f) $-2(-s + 5)$

g) $-5(-t - 8)$ h) $-9(9 - w)$

13. **Assessment Focus** Which pairs of expressions are equivalent?
Explain your reasoning.

a) $2x + 20$ and $2(x + 20)$

b) $3x + 7$ and $10x$

c) $6 + 2t$ and $2(t + 3)$

d) $9 + x$ and $x + 9$

14. There are 15 players on the Grade 8 baseball team. Each player needs a baseball cap and a team jersey.
A team jersey costs \$25. A baseball cap costs \$14.

a) Write 2 different expressions to find the cost of supplying the team with caps and jerseys.

b) Evaluate each expression.
Which expression did you find easier to evaluate? Explain.

15. Five friends go to the movies.
They each pay \$9 to get in, and \$8 for a popcorn and drink combo.

a) Write 2 different expressions to find the total cost of the outing.

b) Evaluate each expression.
Which expression was easier to evaluate? Justify your choice.

16. Match each expression in Column 1 with an equivalent expression in Column 2.

Column 1	Column 2
a) $6(t - 6)$	i) $6t + 36$
b) $-6(t - 6)$	ii) $-6t + 36$
c) $-6(t + 6)$	iii) $-6t - 36$
d) $6(6 + t)$	iv) $6t - 36$

17. **Take It Further**
Harvey won some money on a scratch-and-win ticket. Then, he won a \$2 bonus. When he arrived at the counter, he noticed that he had also won a “triple your winnings” ticket. As Harvey was cashing in his prize, the cashier told him he was the 100th customer, so his total winnings were automatically doubled.
Write two algebraic expressions to describe Harvey’s winnings.

18. **Take It Further**

a) Expand.

i) $7(5 + y - 2)$

ii) $-3(-t + 8 - 3)$

iii) $-8(-9 + s + 5)$

iv) $12(-10 - p + 7)$

b) Choose an expression in part a.
How many different ways can you expand the expression?
Show your work.

19. **Take It Further** Expand.

a) $2(7 + b + c)$ b) $11(-6 + e - f)$

c) $-(-r + s - 8)$ d) $-10(-6 - v - w)$

e) $5(j - 15 - k)$ f) $-4(-g + 12 - h)$

Reflect

How did your knowledge of operations with integers help you in this lesson?

6.5 Solving Equations Involving the Distributive Property

Focus Apply the distributive property to solve an equation.

The distributive property is needed to solve some algebraic equations.

Investigate

Work with a partner.

➤ Alison thought of her favourite number.
She subtracted 2.
Then Alison multiplied the difference by 5.
The product was 60.
What is Alison's favourite number?

Use any strategy to solve the problem.

➤ Write a similar number problem.
Trade problems with another pair of classmates.
Write an algebraic equation, then solve it.

Reflect & Share

Compare your strategy and answer with the same pair of classmates.
Are the equations the same?
How can you check the solution is correct?
How did you solve your classmates' problem?

Connect

These examples show how we can use the distributive property to solve some algebraic equations.

Example 1

John and Lorraine are landscaping their yard. They are buying pyramidal cedars that cost \$12 each. John and Lorraine need 11 cedars to shade their patio on two adjacent sides. They would like to purchase as many more cedars as they can for the far end of their lot. John and Lorraine have \$336 to buy cedars. How many more cedars can they buy?

a) Write an equation that models this problem.
b) Solve the equation.
c) Verify the solution.

A Solution

a) Let e represent how many more cedars John and Lorraine can buy.
Then they will buy a total of $(e + 11)$ cedars. Since the cedars are \$12 each, an equation is: $12(e + 11) = 336$

b)
$$\begin{aligned} 12(e + 11) &= 336 && \text{Use the distributive property to remove the brackets.} \\ 12(e) + 12(11) &= 336 \\ 12e + 132 &= 336 \\ 12e + 132 - 132 &= 336 - 132 \\ 12e &= 204 \\ \frac{12e}{12} &= \frac{204}{12} && \text{Use a calculator.} \\ e &= 17 \end{aligned}$$

John and Lorraine can buy 17 more cedars.

c) To verify the solution, substitute $e = 17$ into $12(e + 11) = 336$.

$$\begin{aligned} \text{Left side} &= 12(e + 11) & \text{Right side} &= 336 \\ &= 12(17 + 11) \\ &= 12(28) \\ &= 336 \end{aligned}$$

Since the left side equals the right side, $e = 17$ is correct.

Example 2

Solve: $14 = 3(x + 4)$
Verify the solution.

A Solution

$14 = 3(x + 4)$
Expand.

$$\begin{aligned} 14 &= 3(x + 4) \\ 14 &= 3(x) + (3)(4) \\ 14 &= 3x + 12 \\ 14 - 12 &= 3x + 12 - 12 \\ 2 &= 3x \\ \frac{2}{3} &= \frac{3x}{3} \\ \frac{2}{3} &= x \\ x &= \frac{2}{3} \end{aligned}$$

To verify the solution, substitute $x = \frac{2}{3}$ into $14 = 3(x + 4)$.

Left side = 14   Right side $= 3(x + 4)$

$$\begin{aligned} &= 3(\tfrac{2}{3} + 4) \\ &= 3(\tfrac{2}{3} + \tfrac{12}{3}) \\ &= 3(\tfrac{14}{3}) \\ &= 14 \end{aligned}$$

Since the left side equals the right side, $x = \frac{2}{3}$ is correct.

Discuss the ideas

1. How could you solve the equation in *Example 1* without using the distributive property?
   What is the first step in the solution?
2. Why would it be more difficult to use this method to solve the equation in *Example 2*?
3. For the solution in *Example 2*, could you use the decimal form of the fraction to verify the solution?
   Justify your answer.

Practice

Check

4. Solve each equation using the distributive property.
Verify the solution.

a) $3(x + 5) = 36$
b) $4(p - 6) = 36$
c) $5(y + 2) = 25$
d) $10(a + 8) = 30$

5. Solve each equation.
Verify the solution.

a) $-2(a + 4) = 18$
b) $-3(r - 5) = -27$
c) $7(-y + 2) = 28$
d) $-6(c - 9) = -42$

6. Marc has some hockey cards.
His friend gives him 3 more cards.
Marc says that if he now doubles the number of cards he has, he will have 20 cards. How many cards did Marc start with?

a) Choose a variable to represent the number of cards Marc started with. Write an equation to model this problem.
b) Solve the equation using the distributive property.
c) Verify the solution. Explain your thinking in words.

7. A student wrote this equation to solve the problem in question 6:
$2n + 3 = 20$
How would you explain to the student why this is incorrect?

Apply

8. The perimeter of a rectangle is 26 cm.
The rectangle has length 8 cm.
What is the width of the rectangle?

a) Write an equation that can be solved using the distributive property.
b) Solve the equation.
c) Verify the solution.

9. **Assessment Focus** The price of a souvenir T-shirt was reduced by $5. Jason bought 6 T-shirts for his friends. The total cost of the T-shirts, before taxes, was $90. What was the price of a T-shirt before it was reduced?

a) Write an equation to model this problem.
b) Solve the equation.
c) Verify the solution. Explain how you know it is correct.

10. Chuck and 7 friends went to Red Deer's Westerner Days fair. The cost of admission was $6 per person. They each bought an unlimited midway ride ticket. The total cost of admission and rides for Chuck and his friends was $264. What was the price of an unlimited midway ride ticket?

a) Write an equation to model this problem.
b) Solve the equation. Verify the solution.

11. Inge chose an integer. She added 9, then multiplied the sum by -5. The product was 15. Which integer did Inge choose?

a) Write an equation you can use to solve the problem.

b) Solve the equation.

c) Verify the solution.

12. Mario chose an integer. He subtracted 7, then multiplied the difference by -4. The product was 36.
Which integer did Mario choose?

a) Write an equation you can use to solve the problem.

b) Solve the equation.

c) Verify the solution.

13. Kirsten used the distributive property to solve this equation: $8(-x + 3) = 8$

a) Check Kirsten's work.
Is her solution correct?

$$\begin{aligned} 8(-x + 3) &= 8 \\ 8(-x) + 8(3) &= 8 \\ -8x + 24 &= 8 \\ -8x + 24 - 24 &= 8 - 24 \\ -8x &= -16 \\ \frac{-8x}{-8} &= \frac{-16}{8} \\ x &= -2 \end{aligned}$$

b) If your answer is yes, verify the solution. If your answer is no, describe the error, then correct it.

14. Solve each equation using the distributive property. Verify the solution.

a) $-10 = 5(t - 2)$ **b)** $7 = 2(p - 3)$

c) $4(r + 5) = 23$ **d)** $-3(s + 6) = 18$

15. **Take It Further**

Amanda's office has 40 employees. The employees want to have a catered dinner. They have found a company that will provide what they need for \$25 per person. Amanda knows that some people will bring a guest. The company has budgeted \$1500 for this event. How many guests can they invite? Assume the price of \$25 includes all taxes.

a) Write an equation for this problem.

b) Solve the equation.

c) Verify the solution.

16. **Take It Further**

Glenn used the equation $7(n - 2) = 42$ to solve a word problem.

a) Create a word problem that can be solved using this equation.

b) Solve the problem.
Verify the solution.

17. **Take It Further**

Solve each equation using the distributive property.
Verify the solution.

a) $7(2 + p - 5) = 14$

b) $8(x - 9 + 7) = -13$

c) $-2(10 - s + 1) = -21$

Reflect

How do you know when to use the distributive property to help you solve an equation? Include examples in your explanation.