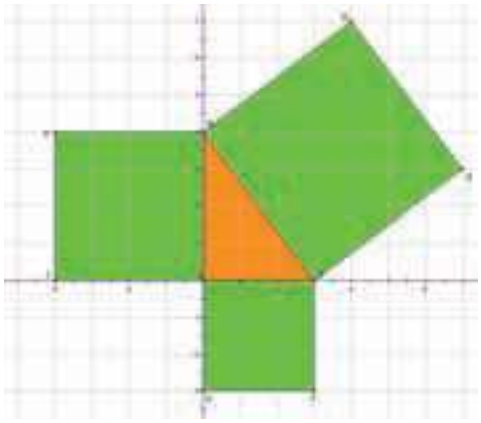
➤ Repeat the previous step, with appropriate rotations, to form squares on AB and AC.



➤ Use the software to find the area of each square. What relationship is shown?

➤ Drag a vertex of the triangle and observe what happens to the area measurements of the squares. How does the geometry software verify the Pythagorean Theorem?

Use the software to investigate "what if" questions.

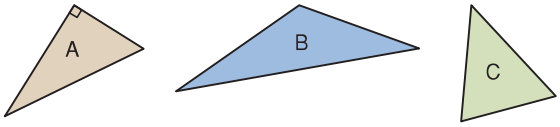
➤ What if the triangle was an acute triangle? Is the relationship still true? Explain.

➤ What if the triangle was an obtuse triangle? Is the relationship still true? Explain.

1.6 Exploring the Pythagorean Theorem

Focus Use the Pythagorean Theorem to identify right triangles.

Look at these triangles.
Which triangle is a right triangle? How do you know?
Which triangle is an obtuse triangle? An acute triangle?
How did you decide?



Investigate

Work on your own.
You will need grid paper and a protractor.

➤ Draw an obtuse triangle.
Draw a square on each side.
Find the area and side length of each square.

➤ Draw an acute triangle.
Draw a square on each side.
Find the area and side length of each square.

➤ Record your results in a table.

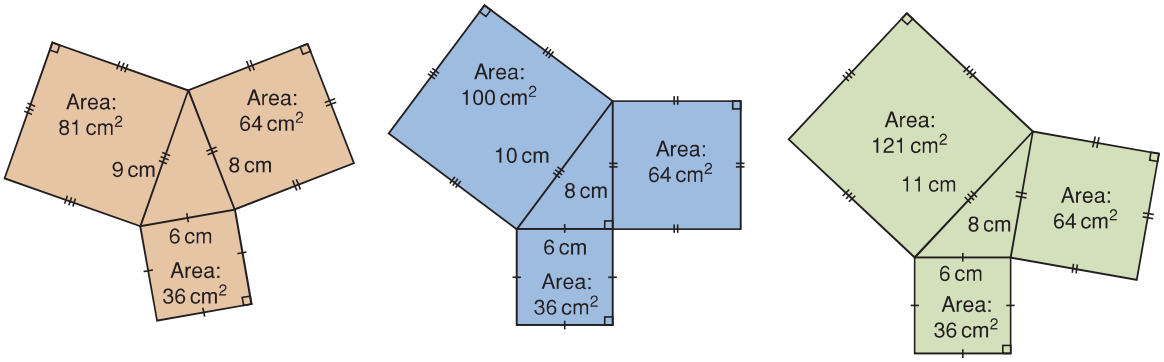
	Area of Square on Shortest Side	Length of Shortest Side	Area of Square on Second Side	Length of Second Side	Area of Square on Longest Side	Length of Longest Side
Obtuse Triangle						
Acute Triangle						

Reflect & Share

Compare your results with those of 3 other classmates.
What relationship do you see among the areas of the squares on the sides of an obtuse triangle?
What relationship do you see among the areas of the squares on the sides of an acute triangle?
How could these relationships help you identify the type of triangle when you know the lengths of its sides?

Connect

Here are an acute triangle, a right triangle, and an obtuse triangle, with squares drawn on the sides of each triangle.



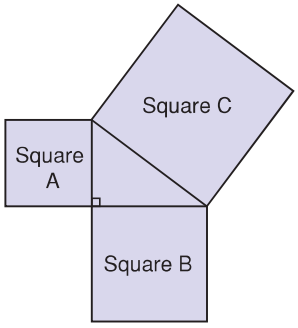
For each triangle, compare the sum of the areas of the two smaller squares to the area of the largest square.

Triangle	Sum of Areas of Two Smaller Squares	Area of Largest Square
Acute	36 + 64 = 100	81
Right	36 + 64 = 100	100
Obtuse	36 + 64 = 100	121

Notice that the Pythagorean Theorem is true for the right triangle only.

We can use these results to identify whether a triangle is a right triangle.
If Area of square A + Area of square B = Area of square C,
then the triangle is a right triangle.

If Area of square A + Area of square B ≠ Area of square C,
then the triangle is not a right triangle.



The symbol ≠ means "does not equal."

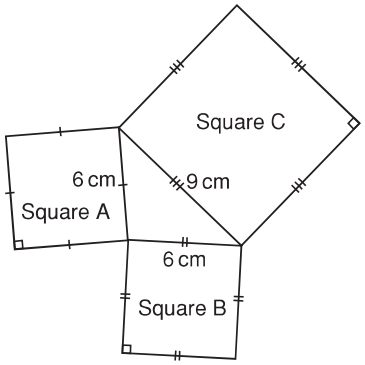
Example 1

Determine whether each triangle with the given side lengths is a right triangle.

a) 6 cm, 6 cm, 9 cm **b)** 7 cm, 24 cm, 25 cm

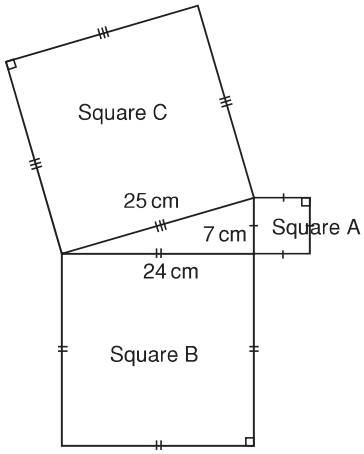
A Solution

a) Sketch a triangle with a square on each side.
The square with the longest side is square C.



Area of square A $= 6 \text{ cm} \times 6 \text{ cm}$
$= 36 \text{ cm}^2$
Area of square B $= 6 \text{ cm} \times 6 \text{ cm}$
$= 36 \text{ cm}^2$
Area of square C $= 9 \text{ cm} \times 9 \text{ cm}$
$= 81 \text{ cm}^2$
Area of square A + Area of square B $= 36 \text{ cm}^2 + 36 \text{ cm}^2$
$= 72 \text{ cm}^2$
Since $72 \text{ cm}^2 \neq 81 \text{ cm}^2$, the triangle is not a right triangle.

b) Sketch a triangle with a square on each side.
The square with the longest side is square C.



Area of square A $= 7 \text{ cm} \times 7 \text{ cm}$
$= 49 \text{ cm}^2$
Area of square B $= 24 \text{ cm} \times 24 \text{ cm}$
$= 576 \text{ cm}^2$
Area of square C $= 25 \text{ cm} \times 25 \text{ cm}$
$= 625 \text{ cm}^2$
Area of square A + Area of square B $= 49 \text{ cm}^2 + 576 \text{ cm}^2$
$= 625 \text{ cm}^2$
Since $625 \text{ cm}^2 = 625 \text{ cm}^2$, the triangle is a right triangle.

A set of 3 whole numbers that satisfies the Pythagorean Theorem is called a **Pythagorean triple**.
For example, 3-4-5 is a Pythagorean triple because $3^2 + 4^2 = 5^2$.

From part b of *Example 1*, a triangle with side lengths 7 cm, 24 cm, and 25 cm is a right triangle.
So, 7-24-25 is a Pythagorean triple.

Example 2

Which of these sets of numbers is a Pythagorean triple?
How do you know?

a) 8, 15, 18 **b)** 11, 60, 61

A Solution

Suppose each set of numbers represents the side lengths of a triangle.
When the set of numbers satisfies the Pythagorean Theorem,
the set is a Pythagorean triple.

a) Check:

Does $8^2 + 15^2 = 18^2$?

L.S. $= 8^2 + 15^2$
$= 64 + 225$
$= 289$

R.S. $= 18^2$
$= 324$

Since $289 \neq 324$, 8-15-18 is not a Pythagorean triple.

b) Check:

Does $11^2 + 60^2 = 61^2$?

L.S. $= 11^2 + 60^2$
$= 121 + 3600$
$= 3721$

R.S. $= 61^2$
$= 3721$

Since $3721 = 3721$, 11-60-61 is a Pythagorean triple.

Discuss the ideas

1. Suppose a square has been drawn on each side of a triangle.
   How can you tell whether the triangle is obtuse?
   How can you tell whether the triangle is acute?
2. Can a right triangle have a hypotenuse of length $\sqrt{65}$ units?

Math Link

History

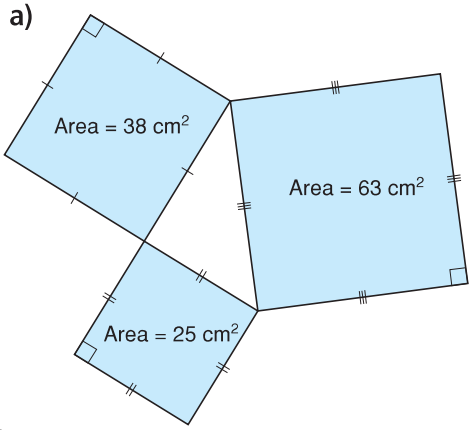
"Numbers Rule the Universe!" That was the belief held by a group of mathematicians called the Brotherhood of Pythagoreans. Their power and influence became so strong that fearful politicians forced them to disband. Nevertheless, they continued to meet in secret and teach Pythagoras' ideas.

Practice

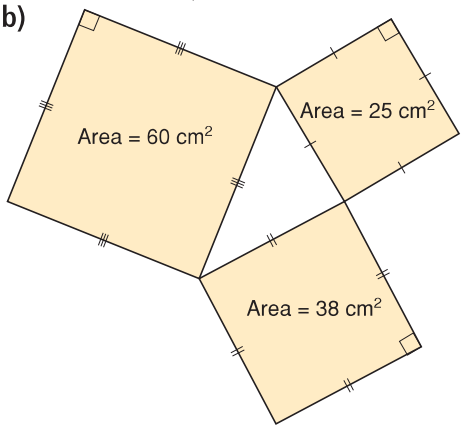
Check

3. The area of the square on each side of a triangle is given. Is the triangle a right triangle? How do you know?

a)

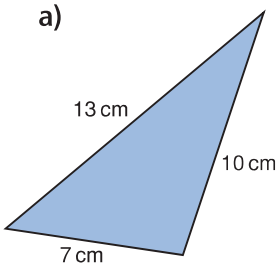


b)

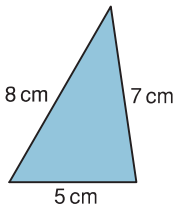


4. Which of these triangles appears to be a right triangle? Determine whether each triangle is a right triangle.
Justify your answers.

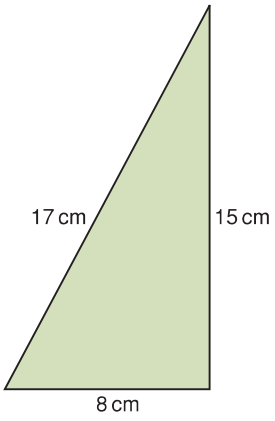
a)



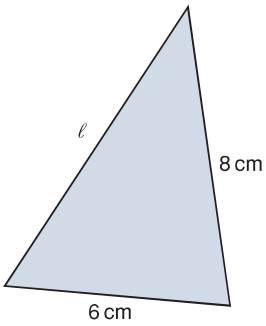
b)



c)



5. Look at the triangle below.
Can the Pythagorean Theorem be used to find the length of the side labelled with a variable? Why or why not?



Apply

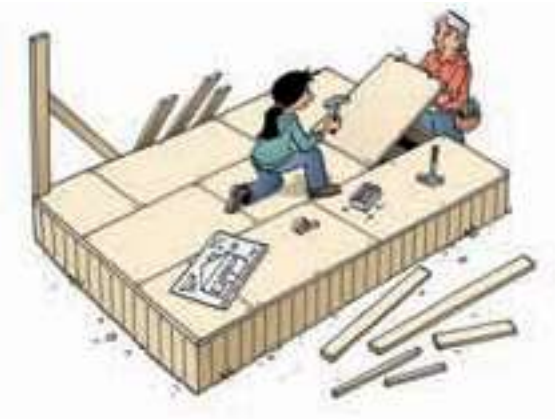
6. Determine whether a triangle with each set of side lengths is a right triangle.
Justify your answers.

a) 16 cm, 30 cm, 34 cm
b) 8 cm, 10 cm, 12 cm
c) 20 m, 25 m, 15 m
d) 28 m, 53 m, 45 m
e) 17 mm, 14 mm, 5 mm
f) 30 mm, 9 mm, 25 mm
g) 9 cm, 9 cm, 15 cm
h) 10 cm, 26 cm, 24 cm

7. Which sets of numbers below are Pythagorean triples?
How did you decide?

a) 16, 30, 34
b) 6, 8, 9
c) 15, 39, 36
d) 16, 65, 63
e) 9, 30, 35
f) 40, 42, 58

8. An elder and his granddaughter, Kashala, are laying a plywood floor in a cabin.
The floor is rectangular, with side lengths 9 m and 12 m.
Kashala measures the diagonal of the floor as 15 m.
Is the angle between the two sides a right angle?
Justify your answer.



9. A triangle has side lengths 6 cm, 7 cm, and $\sqrt{13}$ cm.
Is this triangle a right triangle?
Do these side lengths form a Pythagorean triple? Explain.

10. **Assessment Focus**
May Lin uses a ruler and compass to construct a triangle with side lengths 3 cm, 5 cm, and 7 cm. Before May Lin constructs the triangle, how can she tell if the triangle will be a right triangle?
Explain.

11. Look at the Pythagorean triples below.

3, 4, 5 6, 8, 10
12, 16, 20 15, 20, 25
21, 28, 35

a) Each set of numbers represents the side lengths of a right triangle.
What are the lengths of the legs?
What is the length of the hypotenuse?
b) Describe any pattern you see in the Pythagorean triples.
c) Use a pattern similar to the one you found in part b.
Generate 4 more Pythagorean triples from the triple 5, 12, 13.

12. Two numbers in a Pythagorean triple are given.
Find the third number.
How did you find out?

a) 14, 48, ☐ **b)** 32, 24, ☐
c) 12, 37, ☐ **d)** 20, 101, ☐