

# 3.5

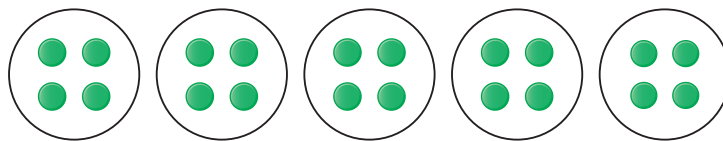
## Dividing Whole Numbers and Fractions

**Focus** Use models to divide proper fractions and whole numbers.

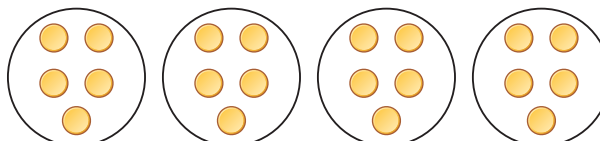
When you first studied division, you learned two ways: sharing and grouping

For example,  $20 \div 5$  can be thought of as:

- Sharing 20 items equally among 5 sets  
There are 4 items in each set.



- Grouping 20 items into sets of 5  
There are 4 sets.



Recall that multiplication and division are inverse operations.

We know:  $20 \div 5 = 4$

What related multiplication facts do you know?

### Investigate

Work with a partner.

You will need scissors.

Your teacher will give you 2 large copies of this diagram.

Square A models the whole number 1.

Figure B represents  $\frac{3}{4}$  of Square A.

Which number does Rectangle C model?

Use paper cutting.

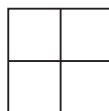
- Use one copy of the diagram.

Find:  $2 \div \frac{1}{4}$

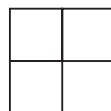
- Use the other copy of the diagram.

Find:  $2 \div \frac{3}{4}$

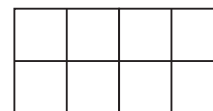
Illustrate your answers.



A



B



C

### Reflect & Share

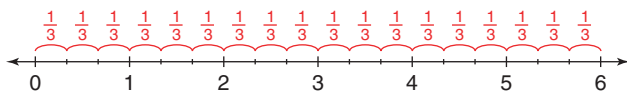
Compare your answers with those of another pair of classmates.

Did you deal with the leftover pieces the same way?

If not, explain your method to your classmates.

## Connect

- We can use a number line to divide a whole number by a fraction. To find how many thirds are in 6, divide 6 into thirds.

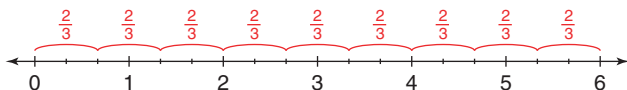


There are 18 thirds in 6.

Write this as a division equation.

$$6 \div \frac{1}{3} = 18$$

- Use the same number line to find how many two-thirds are in 6.



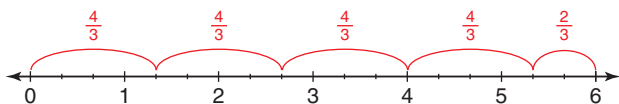
Arrange 18 thirds into groups of two-thirds.

There are 9 groups of two-thirds.

We write:  $6 \div \frac{2}{3} = 9$

- Use the number line again to find how many four-thirds are in 6; that is,  $6 \div \frac{4}{3}$ .

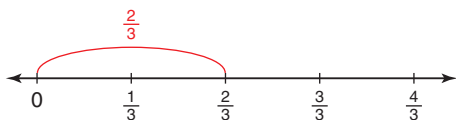
Arrange 18 thirds into groups of four-thirds.



There are 4 groups of 4 thirds.

There are 2 thirds left over.

Think: What fraction of 4 thirds is 2 thirds?



From the number line,  $\frac{2}{3}$  is  $\frac{1}{2}$  of  $\frac{4}{3}$ .

So,  $6 \div \frac{4}{3} = 4\frac{1}{2}$

We can also use a number line to divide a fraction by a whole number. This is illustrated in *Example 1*.

### Example 1

Find each quotient.

- Benny has one-half a litre of milk to pour equally among 3 glasses.  
How much milk should he pour into each glass?
- Chen and Luke equally shared  $\frac{3}{4}$  of a pizza.  
How much of the whole pizza was each person's share?

#### A Solution

- a) Find:  $\frac{1}{2} \div 3$

Think: Share  $\frac{1}{2}$  into 3 equal parts.

Use a number line. Mark  $\frac{1}{2}$  on the line.

Divide the interval 0 to  $\frac{1}{2}$  into 3 equal parts.



$$\frac{1}{2} = \frac{3}{6}$$

Each part is  $\frac{1}{6}$ .

$$\text{So, } \frac{1}{2} \div 3 = \frac{1}{6}$$

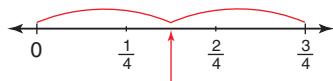
Benny should pour  $\frac{1}{6}$  of a litre of milk into each glass.

- b) Find:  $\frac{3}{4} \div 2$

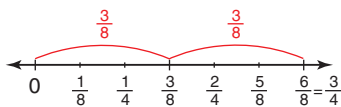
Think: Share  $\frac{3}{4}$  into 2 equal parts.

Use a number line. Mark  $\frac{3}{4}$  on the line.

Divide the interval 0 to  $\frac{3}{4}$  into 2 equal parts.



To label this point, divide the fourths into eighths.



$$\frac{3}{4} = \frac{6}{8}$$

Each part is  $\frac{3}{8}$ .

$$\text{So, } \frac{3}{4} \div 2 = \frac{3}{8}$$

Each person's share was  $\frac{3}{8}$  of the pizza.

Sometimes, when we divide fractions and whole numbers, there is a remainder.  
This remainder is written as a fraction of the divisor.

## Example 2

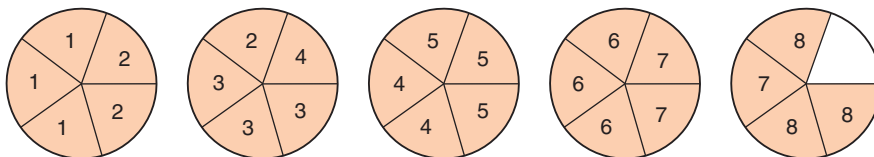
Use a model to divide:  $5 \div \frac{3}{5}$

### A Solution

$$5 \div \frac{3}{5}$$

Think: How many  $\frac{3}{5}$  are in 5 wholes?

Use fraction circles in fifths to model 5.



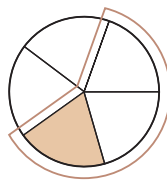
Count groups of three-fifths.

There are 8 groups of three-fifths.

There is 1 fifth left over.

The diagram shows that  $\frac{1}{5}$  is  $\frac{1}{3}$  of  $\frac{3}{5}$ .

$$\text{So, } 5 \div \frac{3}{5} = 8\frac{1}{3}$$



## Discuss the ideas

- In *Example 1a*, the quotient is less than the dividend; that is,  $\frac{1}{6} < \frac{1}{2}$ .  
In *Example 2*, the quotient is greater than the dividend; that is,  $8\frac{1}{3} > 5$ .  
Why do you think this happens?
- How can you use multiplication to check the quotient?

## Practice

### Check

3. Use each picture to find the quotient.  
Write the division equation each time.

a)  $4 \div \frac{1}{3}$



b)  $3 \div \frac{1}{6}$



c)  $4 \div \frac{2}{3}$



d)  $3 \div \frac{3}{5}$



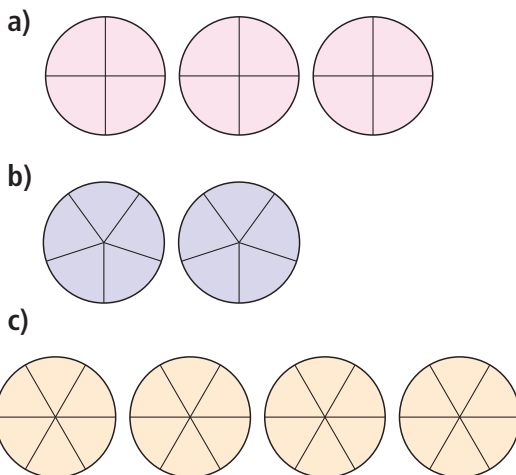
- 4.** Use fraction circles to find:  $4 \div \frac{5}{6}$
- Use fraction circles to model 4. How did you know which fraction circles to use?
  - How many groups of five-sixths are in 4? What is the remainder?
  - What fraction of  $\frac{5}{6}$  does the remainder represent?
  - Write the division equation.

- 5.** Ioana wants to spend  $\frac{4}{5}$  of an hour studying each subject. She has 4 h to study. How many subjects can she study?

- 6.** Use fraction circles. Find each quotient.
- $2 \div \frac{1}{2}$
  - $3 \div \frac{1}{3}$
  - $4 \div \frac{1}{4}$
  - $2 \div \frac{1}{6}$
  - $3 \div \frac{1}{2}$
  - $6 \div \frac{3}{4}$

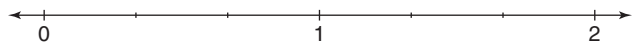
### Apply

- 7.** Which division statement might each picture represent? How many different statements can you write each time? Use fraction circles if they help.

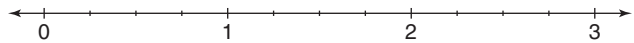


- 8.** Use a number line to find each quotient.

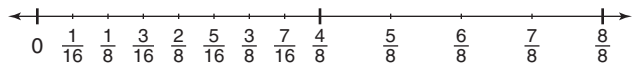
a) i)  $2 \div \frac{1}{3}$       ii)  $2 \div \frac{2}{3}$



b) i)  $3 \div \frac{1}{4}$       ii)  $3 \div \frac{2}{4}$       iii)  $3 \div \frac{3}{4}$



c) i)  $\frac{4}{8} \div 2$       ii)  $\frac{4}{8} \div 4$       iii)  $\frac{4}{8} \div 8$



- 9.** Use a model. Find each quotient.

- $5 \div \frac{2}{3}$
- $4 \div \frac{3}{4}$
- $\frac{1}{2} \div 5$
- $\frac{5}{8} \div 2$

- 10.** Find each quotient. Use number lines to illustrate the answers.

- How many  $\frac{1}{4}$ -size sheets can be cut from 5 sheets of paper?
- How many  $\frac{2}{3}$ -cup servings are in 6 cups of fruit?
- Janelle feeds her cat  $\frac{4}{5}$  of a tin of cat food each day. Janelle has 12 tins of cat food. How many days' supply of cat food does Janelle have?



**11.** Find each quotient. Use number lines to illustrate the answers.

- a) Three-quarters of a whole pizza is shared equally among 5 people. What fraction of the whole pizza does each person get?
- b) One-third of a carton of eggs is used to make a large omelette. How many large omelettes can be made from 4 cartons of eggs?
- c) Brandon planted trees for  $\frac{11}{12}$  h. He planted 5 trees. Assume Brandon took the same amount of time to plant each tree. What fraction of an hour did it take to plant 1 tree?

**12. Assessment Focus** Copy these boxes.

$$\square \div \frac{\square}{\square}$$

- a) Write the digits 2, 4, and 6 in the boxes to find as many division expressions as possible.

- b) Which expression in part a has the greatest quotient? The least quotient? How do you know? Show your work.

**13.** Is  $\frac{2}{3} \div 4$  the same as  $4 \div \frac{2}{3}$ ? Use number lines in your explanation.

**14. Take It Further**

- a) Divide:  $8 \div \frac{1}{3}$
- b) Divide:  $\frac{1}{8} \div 3$
- c) Look at the quotients in parts a and b. What do you notice? How can you explain this?

**15. Take It Further** The numbers  $\frac{9}{2}$  and 3 share this property: their difference is equal to their quotient. That is,  $\frac{9}{2} - 3 = \frac{3}{2}$  and  $\frac{9}{2} \div 3 = \frac{3}{2}$ . Find other pairs of numbers with this property. Describe any patterns you see.

## Math Link

### Your World

Usually, a chef measures fractional amounts, such as  $1\frac{1}{2}$  cups. Occasionally, a chef changes a recipe to serve more or fewer people. To do this, the amounts of ingredients are increased or decreased, often by multiplying or dividing fractions, mixed numbers, or whole numbers.



## Reflect

When you divide a whole number by a proper fraction, is the quotient greater than or less than the whole number? Include an example in your explanation.