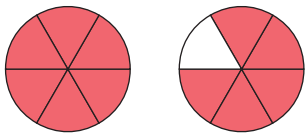
5.6 Adding with Mixed Numbers

Focus Use models, pictures, and symbols to add with mixed numbers.

We have used fraction circles to model and add fractions.
We can also use fraction circles to model and add mixed numbers.
These fraction circles model $1\frac{5}{6}$.



1 whole $\frac{5}{6}$

Explore

Use any materials you want.
A recipe calls for $1\frac{1}{3}$ cups of all-purpose flour and $\frac{5}{6}$ cup of whole-wheat flour.
How much flour is needed altogether?
How can you find out?
Show your work.

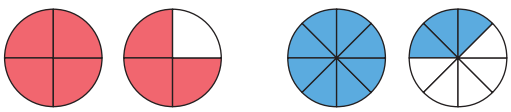
Reflect & Share

Describe your strategy.
Will your strategy work with all mixed numbers?
Test it with $2\frac{1}{3} + \frac{3}{4}$.
Use models or diagrams to justify your strategy.

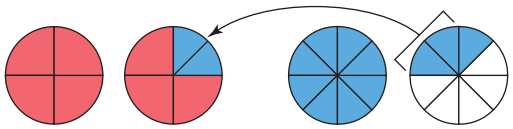
Connect

Use fraction circles to add: $1\frac{3}{4} + 1\frac{3}{8}$

Use fraction circles to model $1\frac{3}{4}$ and $1\frac{3}{8}$.



$1\frac{3}{4}$ $1\frac{3}{8}$



Use eighths to fill the fraction circle for $\frac{3}{4}$.

1 whole and 1 whole and 1 whole and 1 eighth equals 3 wholes and 1 eighth.
So, $1\frac{3}{4} + 1\frac{3}{8} = 3\frac{1}{8}$

To add with mixed numbers, we can:

- Add the fractions and add the whole numbers separately. Or:
- Write each mixed number as an improper fraction, then add.

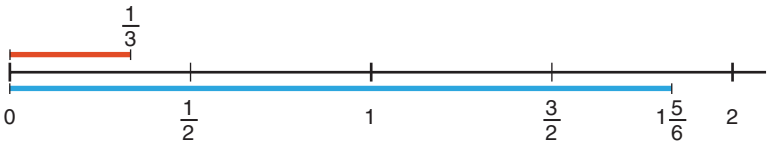
Example

Add: $\frac{1}{3} + 1\frac{5}{6}$

A Solution

$\frac{1}{3} + 1\frac{5}{6}$

Estimate:



$1\frac{5}{6}$ is close to 2.

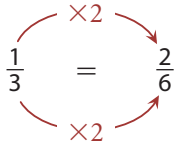
So, $\frac{1}{3} + 1\frac{5}{6} > 2$, but less than $2\frac{1}{3}$

Add the fractions and
the whole number separately.

$\frac{1}{3} + 1\frac{5}{6} = \frac{1}{3} + \frac{5}{6} + 1$

Add the fractions: $\frac{1}{3} + \frac{5}{6}$

Since 6 is a multiple of 3,
use 6 as a common denominator.



$\frac{1}{3} + \frac{5}{6} = \frac{2}{6} + \frac{5}{6}$

$= \frac{7}{6}$

Since 7 > 6, this is an improper fraction.

To write the improper fraction as a mixed number:

$\frac{7}{6} = \frac{6}{6} + \frac{1}{6}$

$= 1 + \frac{1}{6}$

$= 1\frac{1}{6}$

So, $\frac{1}{3} + \frac{5}{6} + 1 = 1\frac{1}{6} + 1$

$= 2\frac{1}{6}$

Then, $\frac{1}{3} + 1\frac{5}{6} = 2\frac{1}{6}$

This is close to the estimate of between 2 and $2\frac{1}{3}$, so the sum is reasonable.

Another Solution

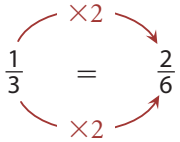
Write the mixed number as an improper fraction, then add.

$1\frac{5}{6} = 1 + \frac{5}{6}$

$= \frac{6}{6} + \frac{5}{6}$

$= \frac{11}{6}$

Since 6 is a multiple of 3, use 6 as a common denominator.



$\frac{1}{3} + 1\frac{5}{6} = \frac{2}{6} + \frac{11}{6}$

$= \frac{13}{6}$

To write the fraction as a mixed number:

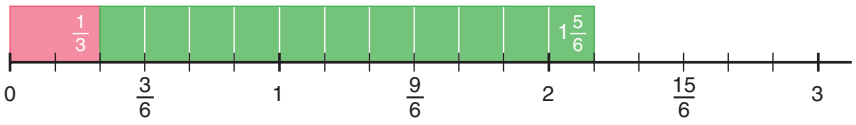
$\frac{13}{6} = \frac{12}{6} + \frac{1}{6}$

$= 2 + \frac{1}{6}$

$= 2\frac{1}{6}$

So, $\frac{1}{3} + 1\frac{5}{6} = 2\frac{1}{6}$

We can model this with a fraction strip on a number line.



Practice

Write all sums in simplest form.

1. Write each mixed number as an improper fraction in simplest form.

a) $1\frac{3}{6}$ **b)** $4\frac{2}{8}$ **c)** $1\frac{3}{4}$ **d)** $3\frac{3}{5}$

2. Write each improper fraction as a mixed number in simplest form.

a) $\frac{17}{5}$ **b)** $\frac{9}{4}$ **c)** $\frac{18}{4}$ **d)** $\frac{28}{6}$

3. Use Pattern Blocks to find each sum.

a) $1\frac{1}{6} + \frac{2}{6}$ **b)** $1\frac{2}{3} + \frac{2}{3}$ **c)** $1\frac{4}{6} + 2\frac{1}{2}$ **d)** $2\frac{1}{3} + 3\frac{5}{6}$

4. Find each sum.

a) $3\frac{2}{3} + 2\frac{1}{3}$ **b)** $1\frac{1}{8} + 3\frac{5}{8}$ **c)** $4\frac{2}{9} + 3\frac{5}{9}$ **d)** $2\frac{3}{5} + 5\frac{4}{5}$

5. Use fraction circles to find each sum.

a) $2\frac{5}{8} + \frac{3}{4}$ **b)** $2\frac{5}{12} + \frac{2}{3}$ **c)** $1\frac{3}{8} + 3\frac{3}{4}$ **d)** $2\frac{2}{5} + 1\frac{7}{10}$

6. We know $\frac{1}{2} + \frac{1}{5} = \frac{7}{10}$.
Use this result to find each sum.
Estimate to check the sum is reasonable.

a) $3\frac{1}{2} + \frac{1}{5}$ **b)** $\frac{1}{2} + 2\frac{1}{5}$ **c)** $3\frac{1}{2} + 2\frac{1}{5}$ **d)** $4\frac{1}{2} + 3\frac{1}{5}$

7. For each pair of numbers, find a common denominator. Then add.

a) $3\frac{1}{3} + \frac{1}{4}$ **b)** $\frac{1}{2} + 1\frac{9}{10}$ **c)** $\frac{3}{4} + 2\frac{3}{5}$ **d)** $\frac{3}{7} + 2\frac{1}{2}$
e) $4\frac{7}{8} + 1\frac{2}{3}$ **f)** $2\frac{3}{5} + 2\frac{2}{3}$ **g)** $5\frac{2}{5} + 1\frac{7}{8}$ **h)** $3\frac{5}{6} + 2\frac{1}{4}$

8. Two students, Galen and Mai, worked on a project.
Galen worked for $3\frac{2}{3}$ h.
Mai worked for $2\frac{4}{5}$ h.
What was the total time spent on the project?

9. **Assessment Focus** Joseph used $1\frac{3}{8}$ cans of paint to paint his room. Juntia used $2\frac{1}{4}$ cans to paint her room.

a) Estimate how many cans of paint were used in all.
b) Calculate how many cans of paint were used.
c) Draw a diagram to model your calculations in part b.

10. A recipe for punch calls for $2\frac{2}{3}$ cups of fruit concentrate and $6\frac{3}{4}$ cups of water.
How many cups of punch will the recipe make?
Show your work.

11. Use the fractions $1\frac{3}{5}$ and $2\frac{1}{10}$.

a) Add the fractions and the whole numbers separately.

b) Write each mixed number as an improper fraction.

c) Add the improper fractions.

d) Which method was easier: adding the mixed numbers or adding the improper fractions? Why do you think so? When would you use each method?

12. An auto mechanic completed 2 jobs before lunch.
The jobs took $2\frac{2}{3}$ h and $1\frac{3}{4}$ h.
How many hours did it take the mechanic to complete the 2 jobs?

13. **Take It Further** Replace the $\square$ with an improper fraction or mixed number to make this equation true.
$3\frac{3}{5} + \square = 5$
Find as many answers as you can.
Draw diagrams to represent your thinking.

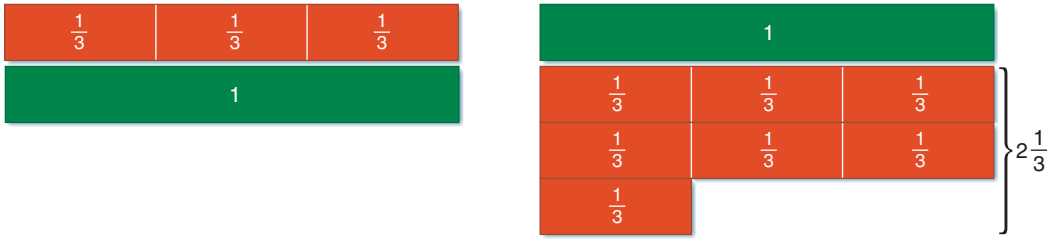
Reflect

How is adding a mixed number and a fraction like adding two fractions?
How is it different?
Use examples to explain.

5.7 Subtracting with Mixed Numbers

Focus Use models, pictures, and symbols to subtract with mixed numbers.

We can use Cuisenaire rods to model fractions and mixed numbers.
Suppose the dark green rod is 1 whole, then the red rod is $\frac{1}{3}$.
So, seven red rods is $\frac{7}{3}$, or $2\frac{1}{3}$.



Explore

Use any materials you want.
A bicycle shop closed for lunch for $1\frac{2}{3}$ h on Monday and for $\frac{3}{4}$ h on Tuesday.
How much longer was the shop closed for lunch on Monday than on Tuesday?
How can you find out? Show your work.

Reflect & Share

Describe your strategy.
Will your strategy work with all mixed numbers?
Test it with $2\frac{1}{4} - \frac{3}{8}$.
Use models or diagrams to justify your strategy.

Connect

Use Cuisenaire rods to subtract: $1\frac{1}{2} - \frac{3}{4}$

Use Cuisenaire rods to model $1\frac{1}{2}$ and $\frac{3}{4}$.

Let the brown rod represent 1 whole.

Then, the purple rod represents $\frac{1}{2}$ and the red rod represents $\frac{1}{4}$.

Model $1\frac{1}{2}$ with Cuisenaire rods.

Model $\frac{3}{4}$ with Cuisenaire rods.

Place the rods for $\frac{3}{4}$ above the rods for $1\frac{1}{2}$, so they align at the right.

Find a rod equal to the difference in their lengths.
The difference is equal to the dark green rod.

The dark green rod represents $\frac{3}{4}$ of the brown rod.
So, $1\frac{1}{2} - \frac{3}{4} = \frac{3}{4}$
To subtract with mixed numbers, we can:

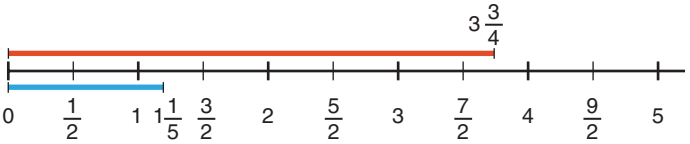
- Subtract the fractions and subtract the whole numbers separately. Or:
- Write each mixed number as an improper fraction, then subtract.

Example

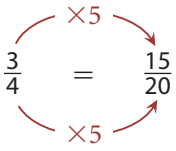
Subtract.
a) $3\frac{3}{4} - 1\frac{1}{5}$ **b)** $3\frac{1}{5} - \frac{3}{4}$
Estimate to check the answer is reasonable.

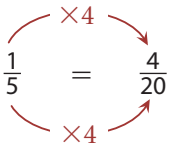
A Solution

a) $3\frac{3}{4} - 1\frac{1}{5}$



Estimate.
$3\frac{3}{4}$ is about 4. $1\frac{1}{5}$ is about 1.
So, $3\frac{3}{4} - 1\frac{1}{5}$ is between 2 and 3.
Subtract the fractions first: $\frac{3}{4} - \frac{1}{5}$
The denominators 4 and 5
have no common factors.
So, a common denominator is: $4 \times 5 = 20$.





$\frac{3}{4} - \frac{1}{5} = \frac{15}{20} - \frac{4}{20}$

$= \frac{11}{20}$

Subtract the whole numbers: $3 - 1 = 2$

Then, $3\frac{3}{4} - 1\frac{1}{5} = 2\frac{11}{20}$

This is close to the estimate of between 2 and 3,

so the answer is reasonable.

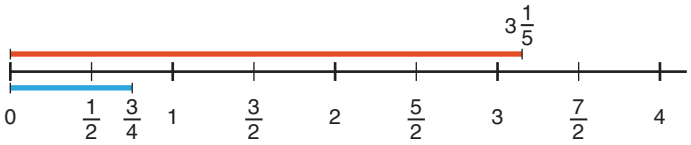
b) $3\frac{1}{5} - \frac{3}{4}$

Estimate.

$3\frac{1}{5}$ is about 3.

$\frac{3}{4}$ is close to 1.

So, $3\frac{1}{5} - \frac{3}{4}$ is about $3 - 1 = 2$.



We cannot subtract the fractions because $\frac{1}{5} < \frac{3}{4}$.

So, write $3\frac{1}{5}$ as an improper fraction.

$3\frac{1}{5} = 3 + \frac{1}{5}$

$= \frac{15}{5} + \frac{1}{5}$

$= \frac{16}{5}$

Another Strategy

We could use fraction circles to subtract.

The denominators have no common factors.

So, a common denominator is: $4 \times 5 = 20$

$\frac{16}{5} \xrightarrow{\times 4} \frac{64}{20}$ $\qquad$ $\frac{3}{4} \xrightarrow{\times 5} \frac{15}{20}$

$\frac{16}{5} - \frac{3}{4} = \frac{64}{20} - \frac{15}{20}$

$= \frac{49}{20}$

$= \frac{40}{20} + \frac{9}{20}$

$= 2 + \frac{9}{20}$

$= 2\frac{9}{20}$

So, $3\frac{1}{5} - \frac{3}{4} = 2\frac{9}{20}$

This is close to the estimate of 2, so the answer is reasonable.

Before we subtract the fraction parts of two mixed numbers,
we must check the fractions to see which is greater.
When the second fraction is greater than the first fraction,
we cannot subtract directly.

Practice

Write all differences in simplest form.

1. Subtract.

a) $2\frac{3}{5} - 1\frac{2}{5}$ **b)** $3\frac{7}{8} - 1\frac{5}{8}$ **c)** $\frac{15}{4} - \frac{3}{4}$ **d)** $\frac{11}{6} - \frac{1}{6}$

2. Subtract. Use Cuisenaire rods.
Sketch diagrams to record your work.

a) $1\frac{2}{3} - \frac{2}{6}$ **b)** $3\frac{1}{2} - 1\frac{2}{4}$ **c)** $3\frac{3}{10} - 2\frac{4}{5}$ **d)** $2\frac{1}{4} - \frac{1}{2}$

3. We know that $\frac{2}{3} - \frac{1}{2} = \frac{1}{6}$.
Use this result to find each difference.
Estimate to check the answer is reasonable.

a) $2\frac{2}{3} - \frac{1}{2}$ **b)** $2\frac{2}{3} - 1\frac{1}{2}$ **c)** $4\frac{2}{3} - 2\frac{1}{2}$ **d)** $5\frac{2}{3} - 1\frac{1}{2}$

4. Estimate, then subtract.

a) $\frac{7}{2} - \frac{5}{4}$ **b)** $\frac{13}{6} - \frac{8}{12}$ **c)** $\frac{5}{4} - \frac{3}{5}$ **d)** $\frac{9}{5} - \frac{1}{2}$

5. **a)** Subtract.

i) $3 - \frac{4}{5}$ **ii)** $4 - \frac{3}{7}$ **iii)** $5 - \frac{5}{6}$ **iv)** $6 - \frac{4}{9}$

b) Which methods did you use in part a?
Explain your choice.

6. For the fractions in each pair of numbers, find a common denominator.
Then subtract.

a) $3\frac{3}{4} - 1\frac{1}{5}$ **b)** $4\frac{9}{10} - 3\frac{1}{2}$ **c)** $3\frac{3}{4} - 1\frac{1}{3}$ **d)** $4\frac{5}{7} - 2\frac{2}{3}$

7. For each pair of mixed numbers below:

a) Subtract the fractions and subtract the whole numbers separately.

b) Write the mixed numbers as improper fractions, then subtract.

c) Which method was easier? Why do you think so?

i) $3\frac{3}{5} - 1\frac{3}{10}$ **ii)** $3\frac{3}{10} - 1\frac{3}{5}$

8. A flask contains $2\frac{1}{2}$ cups of juice.
Ping drinks $\frac{3}{8}$ cup of juice, then Preston drinks $\frac{7}{10}$ cup of juice.
How much juice is in the flask now? Show your work.

9. The running time of a movie is $2\frac{1}{6}$ h.
In the theatre, Jason looks at his watch and sees that $1\frac{1}{4}$ h has passed.
How much longer will the movie run?

10. Subtract.
a) $3\frac{2}{3} - 2\frac{7}{8}$ **b)** $5\frac{1}{2} - 3\frac{7}{9}$ **c)** $4\frac{3}{5} - 1\frac{2}{3}$ **d)** $4\frac{2}{5} - 1\frac{7}{8}$

11. **Assessment Focus** The students in two Grade 7 classes made sandwiches for parents' night.
Mr. Crowe's class used $5\frac{1}{8}$ loaves of bread.
Mme. Boudreau's class used $3\frac{2}{3}$ loaves of bread.
a) Estimate how many more loaves Mr. Crowe's class used.
b) Calculate how many more loaves Mr. Crowe's class used.
c) Draw a diagram to model your calculations in part b.
d) The two classes purchased 10 loaves.
How many loaves were left?

12. **Take It Further** Replace the $\square$ with an improper fraction or mixed number to make this equation true.
$4\frac{1}{8} - \square = 1\frac{1}{2}$
Find as many answers as you can.
Draw diagrams to represent your thinking.

Reflect

You have learned to use improper fractions to subtract mixed numbers.
When is this not the better method? Use an example to explain.