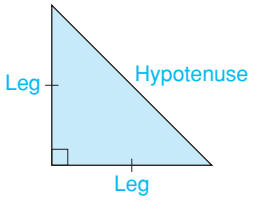
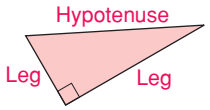
1.5 The Pythagorean Theorem

Focus Discover a relationship among the side lengths of a right triangle.

We can use the properties of a right triangle to find the length of a line segment. A right triangle has two sides that form the right angle. The third side of the right triangle is called the **hypotenuse**. The two shorter sides are called the **legs**.



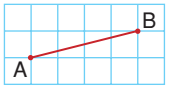
Isosceles right triangle



Scalene right triangle

Investigate

Work on your own.
You will need grid paper, centimetre cubes, and a protractor.



➤ Copy line segment AB.
Draw right triangle ABC that has segment AB as its hypotenuse.
Draw a square on each side of △ABC.
Find the area and side length of each square.

➤ Draw 3 different right triangles, with a square on each side.
Find the area and side length of each square.
Record your results in a table.

See page 18 if you have forgotten how to find the area of a square on a line segment.

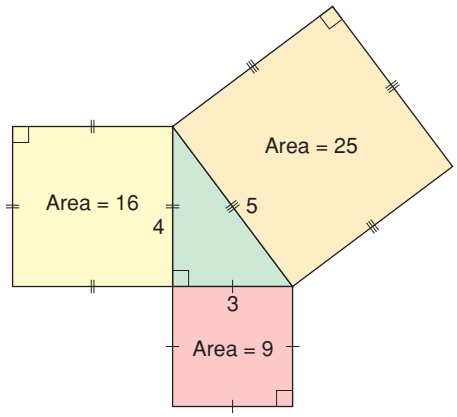
	Area of Square on Leg 1	Length of Leg 1	Area of Square on Leg 2	Length of Leg 2	Area of Square on Hypotenuse	Length of Hypotenuse
Triangle ABC						
Triangle 1						
Triangle 2						
Triangle 3						

Reflect & Share

Compare your results with those of another classmate.
What relationship do you see among the areas of the squares on the sides of a right triangle? How could this relationship help you find the length of a side of a right triangle?

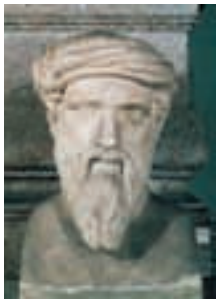
Connect

Here is a right triangle, with a square drawn on each side.



**The area of the square on the hypotenuse is 25.
The areas of the squares on the legs are 9 and 16.**

Notice that: $25 = 9 + 16$
A similar relationship is true for all right triangles.



The Pythagorean Theorem is named for the Greek mathematician, Pythagoras.

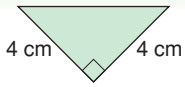
In a right triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the legs.

This relationship is called the **Pythagorean Theorem**.

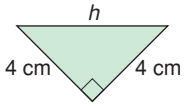
We can use this relationship to find the length of any side of a right triangle, when we know the lengths of the other two sides.

Example 1

Find the length of the hypotenuse.
Give the length to one decimal place.



A Solution

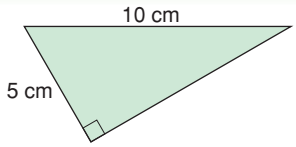


Label the hypotenuse h.
The area of the square on the hypotenuse is h^2.
The areas of the squares on the legs are
$4 \times 4 = 16$ and $4 \times 4 = 16$.
So, $h^2 = 16 + 16$
$= 32$

The area of the square on the hypotenuse is 32.
So, the side length of the square is: $h = \sqrt{32}$
Use a calculator.
$h \doteq 5.6569$
So, the hypotenuse is 5.7 cm to one decimal place.

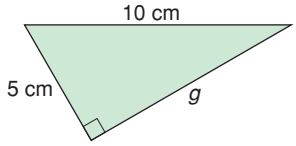
Example 2

Find the unknown length to one decimal place.



A Solution

Label the leg g.
The area of the square on the hypotenuse is $10 \times 10 = 100$.
The areas of the squares on the legs are
g^2 and $5 \times 5 = 25$.
So, $100 = g^2 + 25$
To solve this equation, subtract 25 from each side.
$100 - 25 = g^2 + 25 - 25$
$75 = g^2$
The area of the square on the leg is 75.
So, the side length of the square is: $g = \sqrt{75}$
Use a calculator.
$g \doteq 8.66025$
So, the leg is 8.7 cm to one decimal place.



Discuss the ideas

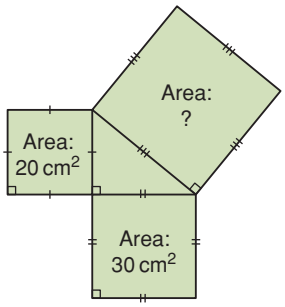
1. How can you identify the hypotenuse of a right triangle?
2. How can you use the Pythagorean Theorem to find the length of the diagonal in a rectangle?

Practice

Check

3. Find the area of the indicated square.

a)



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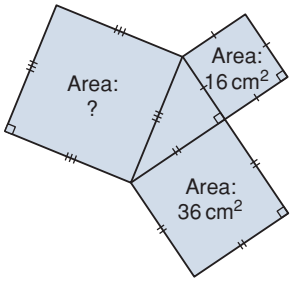
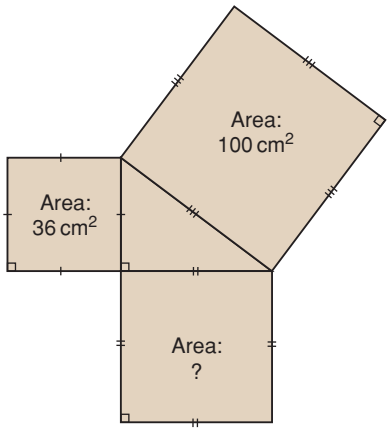


4. Find the area of the indicated square.

a)



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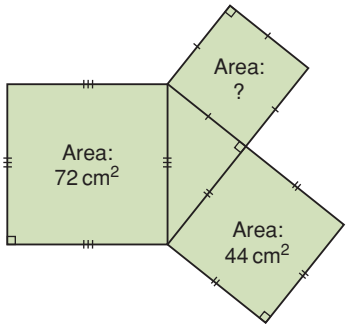
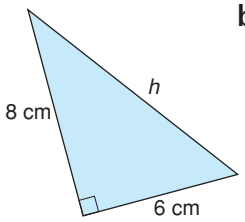


5. Find the length of each hypotenuse. Give your answers to one decimal place where needed.

a)



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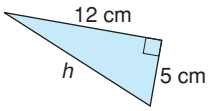

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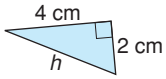
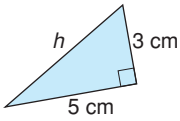
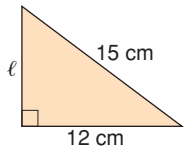


d)



6. Find the length of each leg labelled ℓ. Give your answers to one decimal place where needed.

a)



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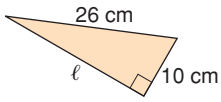

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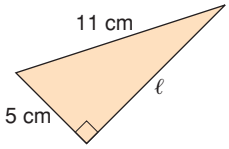

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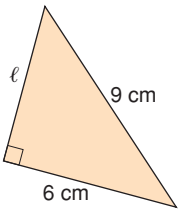
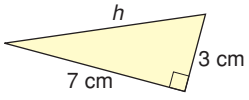


Apply

7. Find the length of each side labelled with a variable. Give your answers to one decimal place where needed.

a)



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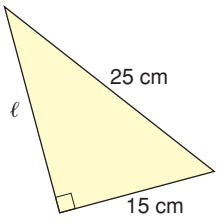

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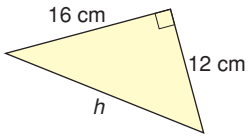
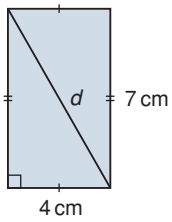


8. Find the length of the diagonal, d, in each rectangle.
Give your answers to two decimal places where needed.

a)



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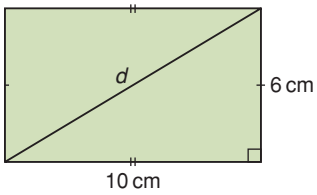

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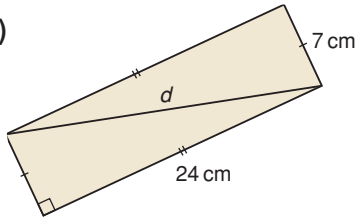
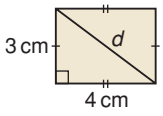


9. Find the length of the diagonal, d, in each rectangle.
What patterns do you notice?
Write to explain.
Use your patterns to draw the next rectangle in the pattern.

a)



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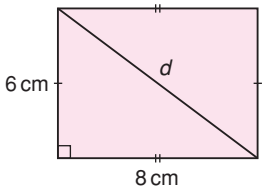

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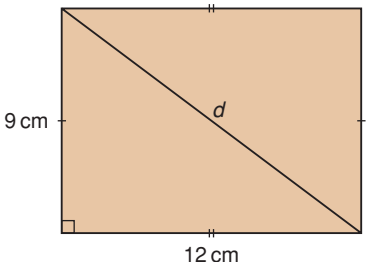
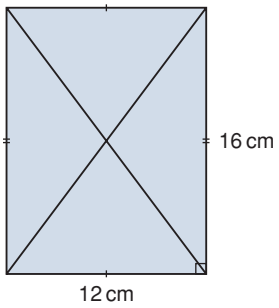


10. Suppose you are given the side lengths of a right triangle.
Which length is the length of the hypotenuse?
Explain how you know.

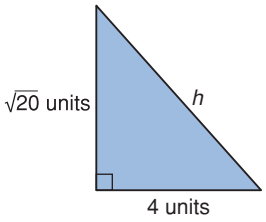
11. Use the rectangle on the right. Explain why the diagonals of a rectangle have the same length.



12. **Assessment Focus** The hypotenuse of a right triangle is $\sqrt{18}$ units.
What are the lengths of the legs of the triangle?
How many different answers can you find?
Sketch a triangle for each answer.
Explain your strategies.

13. Find the length of each side labelled with a variable.

a)



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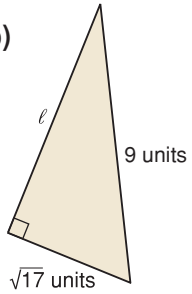

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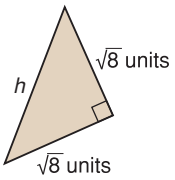


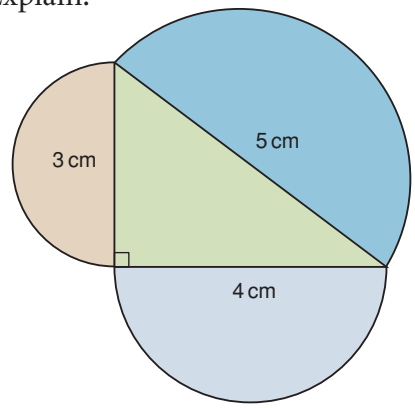
14. Use what you know about the Pythagorean Theorem. On grid paper, draw a line segment with each length. Explain how you did it.

a) $\sqrt{5}$ **b)** $\sqrt{10}$

c) $\sqrt{13}$ **d)** $\sqrt{17}$

15. Take It Further The length of the hypotenuse of a right triangle is 15 cm. The lengths of the legs are whole numbers of centimetres.
Find the sum of the areas of the squares on the legs of the triangle.
What are the lengths of the legs? Show your work.

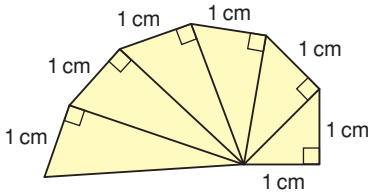
16. Take It Further An artist designed this logo. It is a right triangle with a semicircle drawn on each side of the triangle. Calculate the area of each semicircle. What do you notice? Explain.



17. Take It Further Use grid paper. Draw a right triangle with a hypotenuse with each length.

a) $\sqrt{20}$ units **b)** $\sqrt{89}$ units **c)** $\sqrt{52}$ units

18. Take It Further Your teacher will give you a copy of part of the Wheel of Theodorus.



Theodorus was born about 100 years after Pythagoras. Theodorus used right triangles to create a spiral. Today the spiral is known as the Wheel of Theodorus.

a) Find the length of the hypotenuse in each right triangle.
Label each hypotenuse with its length as a square root.
What pattern do you see?

b) Use a calculator. Write the value of each square root in part a to one decimal place.

c) Use a ruler. Measure the length of each hypotenuse to one decimal place.

d) Compare your answers in parts b and c. What do you notice?

Reflect

Suppose your classmate missed today's lesson. Use an example to show your classmate how to find the length of the third side of a right triangle when you know the lengths of the other two sides.