6.4 Solving Equations Using Algebra

Focus Solve a problem by solving an equation algebraically.

Explore

Solve this problem.
My mother's age is 4 more than 2 times my brother's age.
My mother is 46 years old. How old is my brother?

Reflect & Share

Discuss the strategies you used for finding
the brother's age with those of another pair of classmates.
Did you draw a picture? Did you use tiles? Did you use an equation?
If you did not use an equation, how could you represent
this problem with an equation?

Connect

Solving an equation *using algebra* is often the quickest way to find a solution, especially if the equation involves large numbers.

Recall that when we solve an equation,
we find the value of the variable that makes the equation true.
That is, we find the value of the variable which,
when substituted into the equation,
makes the left side of the equation equal to the right side.

When we solve an equation using algebra, remember
the balance-scales model.
To preserve the equality, always perform the same operation
on both sides of the equation.

Whatever you do to one side of an equation, you do to the other side too.

Example

Three more than two times a number is 27. What is the number?

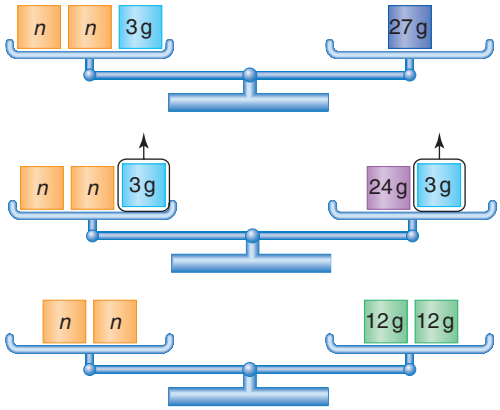
a) Write an equation to represent this problem.

b) Solve the equation. Show the steps.

c) Verify the solution.

A Solution

a) Let n represent the number.
Then two times the number is: $2n$
And, three more than two times the number is: $2n + 3$
The equation is: $2n + 3 = 27$



b) $2n + 3 = 27$
To isolate $2n$, subtract 3 from each side.
$2n + 3 - 3 = 27 - 3$
$2n = 24$
Divide each side by 2.
$\frac{2n}{2} = \frac{24}{2}$
$n = 12$

c) To verify the solution, substitute $n = 12$ into $2n + 3 = 27$.
Left side $= 2n + 3$ Right side $= 27$
$= 2(12) + 3$
$= 24 + 3$
$= 27$
Since the left side equals the right side, $n = 12$ is correct.
The number is 12.

We could also verify by following the instructions in the *Example*.
Start with 12.
Multiply by 2: 24
Add 3: 27

Practice

Solve each equation using algebra.

1. Solve each equation. Verify the solution.
a) $x - 27 = 35$ **b)** $11x = 132$ **c)** $4x + 7 = 75$

2. Write, then solve, an equation to find each number. Verify the solution.
a) Nineteen more than a number is 42.
b) Ten more than three times a number is 25.
c) Fifteen more than four times a number is 63.

3. Five years after Jari's age now doubles, he will be 27. How old is Jari now?
a) Write an equation you can use to solve the problem.
b) Solve the equation. Show the steps. How old is Jari?
c) Verify the solution.

4. Jenny baby-sat on Saturday for $6/h. She was given a $3 bonus.
How many hours did Jenny baby-sit if she was paid $33?

a) Write an equation you can use to solve the problem.

b) Solve the equation. How many hours did Jenny baby-sit?

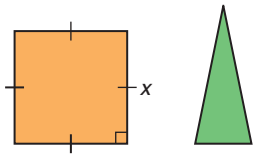
c) Verify the solution.

5. In x weeks and 4 days, the movie *Math-Man IV* will be released.
The movie will be released in 25 days. Find the value of x.

a) Write an equation you can use to solve the problem.

b) Solve the equation. Verify the solution.

6. Look at the square and triangle on the right.
The sum of their perimeters is 56 cm.
The perimeter of the triangle is 24 cm.
What is the side length of the square?



a) Write an equation you can use to find the side length of the square.

b) Solve the equation. Verify the solution.

7. **Assessment Focus** Sunita has $72 in her savings account.
Each week she saves $24. When will Sunita have a total savings of $288?

a) Write an equation you can use to solve the problem.

b) Solve the equation. Show the steps.
When will Sunita have $288 in her savings account?

c) How can you check the answer?

8. **Take It Further** Use the information on the sign to the right.

a) Write a problem that can be solved using an equation.

b) Write the equation, then solve the problem.

c) Show how you could solve the problem without writing an equation.

9. **Take It Further** The nth term of a number pattern is $9n + 1$.
What is the term number for each term value?

a) 154 **b)** 118 **c)** 244

Reflect

What advice would you give someone who is having difficulty solving equations using algebra?

6.5 Using Different Methods to Solve Equations

Focus Decide which method to use to solve an equation.

Recall the methods you have used to solve an equation:

- using algebra tiles
- by inspection
- by systematic trial
- using a balance-scales model
- using algebra

Explore

Lila, Meeka, and Noel are playing darts.
Each player throws 3 darts at the board.
A player's score is the sum of the numbers in the areas the darts land.
This picture shows a score of:
$8 + 20 + 2 = 30$

Write an equation for each problem.
Solve the equation using a method of your choice.

➤ Lila's first two darts scored a total of 12 points.
Lila scored 20 points in the round.
How many points did she score with her third dart?

➤ All three of Meeka's darts landed in the same area.
She scored 63 points. In which area did all her darts land?

➤ Noel's first two darts landed in the same area.
Her third dart was a bull's-eye, scoring 50 points.
She scored a total of 72 points.
In which area did her first two darts land?

Reflect & Share

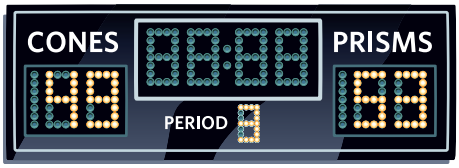
Compare your equations with those of another pair of classmates.
Explain why you chose the method you did to solve them.
Use a different method to solve one of the equations.
Did this method work better for you? Why do you think so?

Connect

We can use any method to solve an equation, as long as the steps we take make sense, and the correct solution is found.

You can always check if the solution is correct by substituting the solution into the original equation.

Example

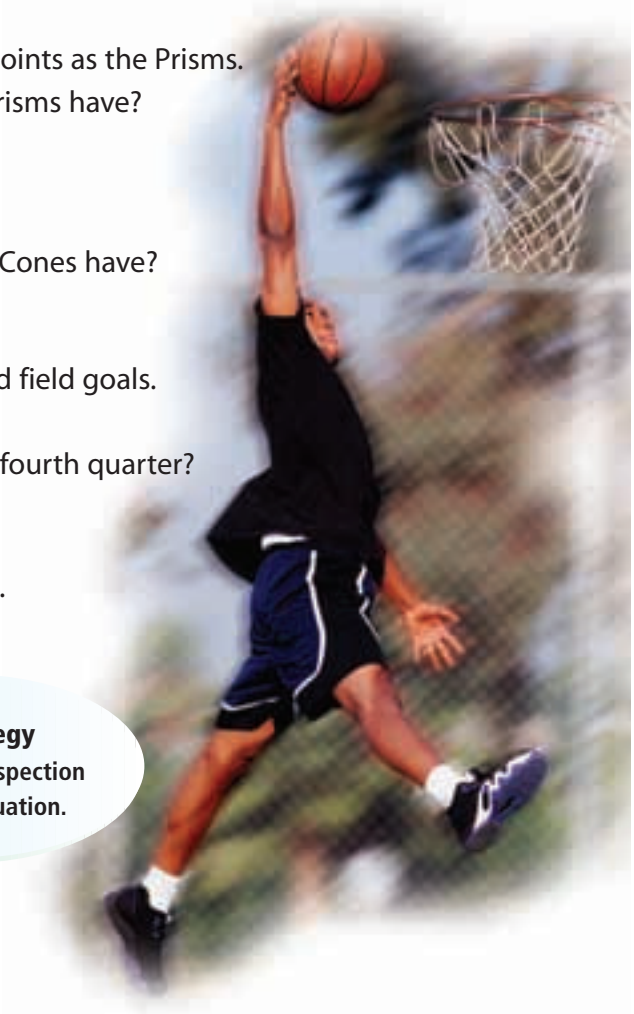


In a basketball game between the Central City Cones and the Park Town Prisms, the lead changed sides many times.
Write, then solve, an equation to solve each problem.

a) Early in the game, the Cones had one-half as many points as the Prisms.
The Cones had 8 points. How many points did the Prisms have?

b) Near the end of the first half, the Cones were 12 points ahead of the Prisms.
The Prisms had 39 points. How many points did the Cones have?

c) The Prisms scored 32 points in the fourth quarter.
Twenty of these points were scored by foul shots and field goals.
The rest of the points were scored by 3-point shots.
How many 3-point shots did the Prisms make in the fourth quarter?



A Solution

a) Let p represent the number of points the Prisms had.
The Cones had one-half as many: $\frac{p}{2}$
The Cones had 8 points.
The equation is: $\frac{p}{2} = 8$
Solve using algebra.
Multiply each side by 2.

$$\frac{p}{2} \times 2 = 8 \times 2$$
$$\frac{2p}{2} = 16$$
$$p = 16$$

The Prisms had 16 points.

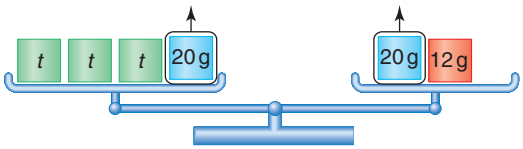
Another Strategy
We could use inspection to solve this equation.

b) Let d represent the number of points the Cones had.
The Prisms had 12 fewer points: $d - 12$
An equation is: $d - 12 = 39$
Solve using systematic trial. We choose a value for d and substitute.
Try $d = 50$. $d - 12 = 50 - 12$
$= 38$
38 is close. Choose a greater value for d.
Try $d = 51$. $d - 12 = 51 - 12$
$= 39$
This is correct. The Cones had 51 points.

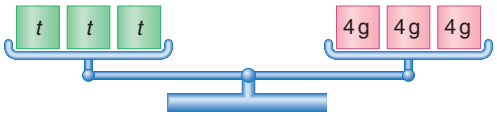
Another Strategy
We could use algebra to solve this equation.

c) Let t represent the number of 3-point shots the Prisms made in the fourth quarter.
So, $3t$ represents the number of points scored by 3-point shots.
The equation is: $3t + 20 = 32$
Use a balance-scales model.

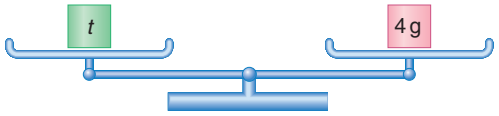
To isolate t, 20 g has to be removed from the left pan.
So, replace 32 g in the right pan with masses of 20 g and 12 g, since 20 g + 12 g = 32 g. Then, remove 20 g from each pan.



Three identical unknown masses remain in the left pan.
12 g remain in the right pan. Replace 12 g with three 4-g masses.



The three unknown masses balance with three 4-g masses.
So, each unknown mass is 4 g.



The Prisms made four 3-point shots in the fourth quarter.

In *Example,* part c, we can solve the equation using algebra to check.

$$3t + 20 = 32$$

$3t + 20 - 20 = 32 - 20$ To isolate $3t$, subtract 20 from each side.

$$3t = 12$$

$\frac{3t}{3} = \frac{12}{3}$ To isolate t, divide each side by 3.

$$t = 4$$

The algebraic solution and the balance-scales solution are the same.
It was much quicker to solve the equation using algebra.

Practice

Use algebra, systematic trial, inspection, algebra tiles, or a balance-scales model to solve each equation.

1. Use algebra to solve each equation. Verify each solution.

a) $\frac{x}{2} = 4$ **b)** $\frac{x}{3} = 7$ **c)** $\frac{x}{4} = 16$ **d)** $\frac{x}{5} = 10$

2. Which method would you choose to solve each equation?
Explain your choice.
Solve each equation using the method of your choice.

a) $x + 5 = 12$ **b)** $x - 5 = 12$ **c)** $\frac{x}{6} = 9$ **d)** $x + 4 = -9$

e) $4x = 36$ **f)** $16x = 112$ **g)** $4x + 2 = 30$ **h)** $8x + 17 = 105$

3. George and Mary collect friendship beads.
George gave Mary 7 beads.
Mary then had 21 beads.
How many friendship beads did Mary have to start with?

a) Write, then solve, an equation you can use to solve this problem.

b) Verify the solution.

4. Jerome baked some cookies.
He shared them among his eight friends.
Each friend had 4 cookies.
Write, then solve, an equation to find how many cookies Jerome baked.

5. Which method do you prefer to use to solve an equation?
Explain. Give an example.

6. **Assessment Focus** Carla has 20 songs downloaded to her MP3 player.
Each month she downloads 8 additional songs.
After how many months will Carla have a total of 92 songs?

a) Use an equation to solve the problem.

b) Which method did you choose to solve the equation?
Explain why you chose this method.

7. Write, then solve, an equation to answer each question.
Verify the solution.
Sheng sorted 37 cans.

a) He divided the cans into 4 equal groups.
He had 5 cans left over.
How many cans were in each group?

b) He divided the cans into 9 equal groups.
He had 10 cans left over.
How many cans were in each group?

8. Write, then solve, an equation to answer each question.
Verify the solution.
At Pascal's Pet Store, a 5-kg bag of dog food costs $10.
The 10-kg bag costs $15.

a) Pascal sold $85 worth of dog food.
He sold four 5-kg bags. How many 10-kg bags did he sell?

b) Pascal sold $140 worth of dog food.
He sold six 10-kg bags. How many 5-kg bags did he sell?

9. **Take It Further** Refer to the dart problem in *Explore*, page 240.

a) Write two more problems using the given information.
For each problem, write an equation you can use to solve your problem. Solve the equation.
Use a different method for each equation.

b) How could a player score 35 points with 3 darts?
Find as many different ways as you can.

Reflect

Talk to a partner.
Tell how you choose the method you use to solve an equation.