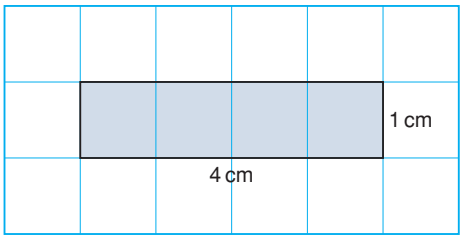
1.1 Square Numbers and Area Models

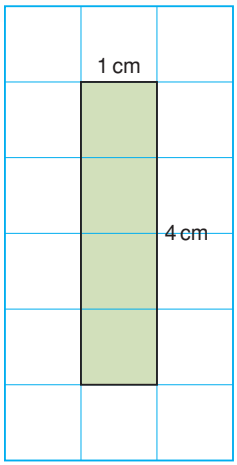
Focus Relate the area of a square and square numbers.

A rectangle is a quadrilateral with 4 right angles.
A square also has 4 right angles.

A rectangle with base 4 cm and height 1 cm
is the same as a rectangle with base 1 cm and height 4 cm.

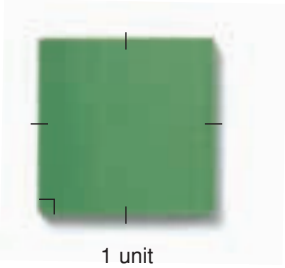


These two rectangles are *congruent*.
Is every square a rectangle?
Is every rectangle a square?



Investigate

Work with a partner.
You will need grid paper and 20 square tiles like this:
Use the tiles to make as many different rectangles
as you can with each area.



4 square units
6 square units
8 square units
9 square units
12 square units
16 square units
20 square units

Draw the rectangles on grid paper.

- For how many areas above were you able to make a square?
- What is the side length of each square you made?
- How is the side length of a square related to its area?

Reflect & Share

Compare your strategies and results with those of another pair of classmates.
Find two areas greater than 20 square units for which you could use tiles to make a square.
How do you know you could make a square for each of these areas?

Connect

When we multiply a number by itself, we *square* the number.
For example: The square of 4 is $4 \times 4 = 16$.
We write: $4 \times 4 = 4^2$
So, $4^2 = 4 \times 4 = 16$
We say: Four squared is sixteen.
16 is a **square number**, or a **perfect square**.

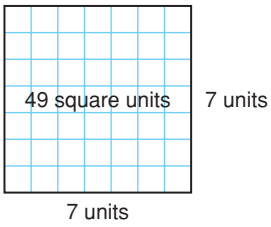
One way to model a square number is to draw a square whose area is equal to the square number.

Example 1

Show that 49 is a square number.
Use a diagram, symbols, and words.

A Solution

Draw a square with area 49 square units.
The side length of the square is 7 units.
Then, $49 = 7 \times 7 = 7^2$
We say: Forty-nine is seven squared.

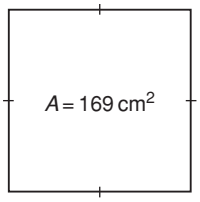


Example 2

A square picture has area 169 cm^2.
Find the perimeter of the picture.

A Solution

The picture is a square with area 169 cm^2.
Find the side length of the square:



Find a number which, when multiplied by itself, gives 169.
$13 \times 13 = 169$
So, the picture has side length 13 cm.

Perimeter is the distance around the picture.
So, $P = 13 \text{ cm} + 13 \text{ cm} + 13 \text{ cm} + 13 \text{ cm}$
$= 52 \text{ cm}$
The perimeter of the picture is 52 cm.

Discuss the ideas

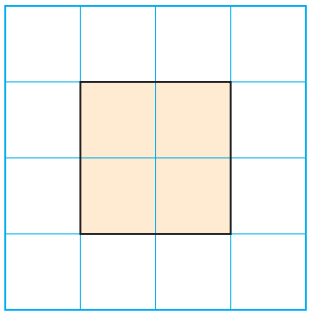
1. Is 1 a square number? How can you tell?
2. Suppose you know the area of a square.
   How can you find its perimeter?
3. Suppose you know the perimeter of a square.
   How can you find its area?

Practice

Check

4. Match each square below to its area.

a)



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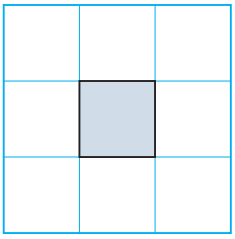
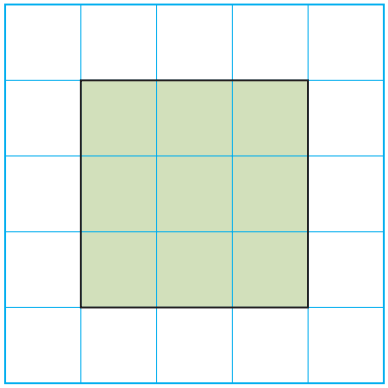


c)



i) 1 unit × 1 unit = 1 square unit
ii) 2 units × 2 units = 4 square units
iii) 3 units × 3 units = 9 square units

5. Find the area of a square with each side length.
   a) 8 units **b)** 10 units **c)** 3 units

6. Use square tiles.
   Make as many different rectangles as you can with area 36 square units.
   Draw your rectangles on grid paper.
   Is 36 a perfect square?
   Justify your answer.

Apply

7. Use square tiles.
   Make as many different rectangles as you can with area 28 square units.
   Draw your rectangles on grid paper.
   Is 28 a perfect square?
   Justify your answer.

8. Show that 25 is a square number.
   Use a diagram, symbols, and words.

9. Show that 12 is not a square number.
   Use a diagram, symbols, and words.

10. Use a diagram to show that each number below is a square number.

a) 1 **b)** 144

c) 121 **d)** 900

11. Find the side length of a square with each area.

a) 100 m^2 **b)** 64 cm^2

c) 81 m^2 **d)** 400 cm^2

12. Which of these numbers is a perfect square?
How do you know?

a) 10 **b)** 50

c) 81 **d)** 20

13. Use 1-cm grid paper.
Draw as many different rectangles as you can with area 64 cm^2.
Find the base and height of each rectangle.
Record the results in a table.

Base (cm)	Height (cm)	Perimeter (cm)

Which rectangle has the least perimeter?
What can you say about this rectangle?

14. I am a square number.
The sum of my digits is 9.
What square numbers might I be?

15. These numbers are not square numbers.
Which two consecutive square numbers is each number between?
Describe the strategy you used.

a) 12 **b)** 40

c) 75 **d)** 200

16. The floor of a large square room has area 144 m^2.

a) Find the length of a side of the room.

b) How much baseboard is needed to go around the room?

c) Each piece of baseboard is 2.5 m long.
How many pieces of baseboard are needed?
What assumptions do you make?

17. A garden has area 400 m^2.
The garden is divided into 16 congruent square plots.
Sketch a diagram of the garden.
What is the side length of each plot?

18. **Assessment Focus** Which whole numbers between 50 and 200 are perfect squares?
Explain how you know.

19. Lee is planning to fence a square kennel for her dog.
Its area must be less than 60 m^2.
a) Sketch a diagram of the kennel.
b) What is the kennel's greatest possible area?
c) Find the side length of the kennel.
d) How much fencing is needed?
e) One metre of fencing costs $10.00.
What is the cost of the fencing?
What assumptions do you make?

20. **Take It Further** Devon has a piece of poster board 45 cm by 20 cm.
His teacher challenges him to cut the board into parts, then rearrange the parts to form a square.
a) What is the side length of the square?
b) What are the fewest cuts Devon could have made?
Explain.

21. **Take It Further** The digital root of a number is the result of adding the digits of the number until a single-digit number is reached.
For example, to find the digital root of 147:
$1 + 4 + 7 = 12$ and $1 + 2 = 3$
a) Find the digital roots of the first 15 square numbers.
What do you notice?
b) What can you say about the digital root of a square number?
c) Use your results in part b.
Which of these numbers might be square numbers?
i) 440 **ii)** 2809 **iii)** 3008
iv) 4225 **v)** 625

Reflect

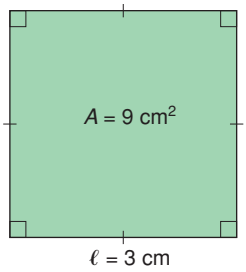
Use diagrams to explain why 24 is not a square number but 25 is a square number.

1.4 Estimating Square Roots

Focus Develop strategies for estimating a square root.

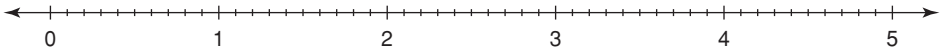
You know that a square root of a given number is a number which, when multiplied by itself, results in the given number.
For example, $\sqrt{9} = \sqrt{3 \times 3}$
$= 3$

You also know that the square root of a number is the side length of a square with area that is equal to that number.
For example, $\sqrt{9} = 3$



Investigate

Work with a partner.
Use a copy of the number line below.
Place each square root on the number line to show its approximate value: $\sqrt{2}$, $\sqrt{5}$, $\sqrt{11}$, $\sqrt{18}$, $\sqrt{24}$
Write each estimated square root as a decimal.
Use grid paper if it helps.



Reflect & Share

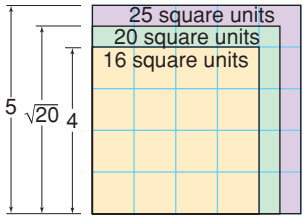
Compare your answers with those of another pair of classmates.
What strategies did you use to estimate the square roots?
How could you use a calculator to check your estimates?

Connect

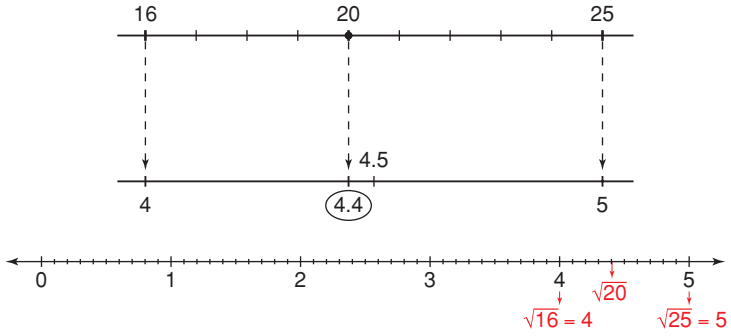
Here is one way to estimate the value of $\sqrt{20}$:

- 25 is the square number closest to 20, but greater than 20.
  On grid paper, draw a square with area 25.
  Its side length is: $\sqrt{25} = 5$
- 16 is the square number closest to 20, but less than 20.
  Draw a square with area 16.
  Its side length is: $\sqrt{16} = 4$

Draw the squares so they overlap.



A square with area 20 lies between these two squares.
Its side length is $\sqrt{20}$.
20 is between 16 and 25, but closer to 16.
So, $\sqrt{20}$ is between $\sqrt{16}$ and $\sqrt{25}$, but closer to $\sqrt{16}$.
So, $\sqrt{20}$ is between 4 and 5, but closer to 4.
An estimate of $\sqrt{20}$ is 4.4 to one decimal place.



Example 1

Which whole number is $\sqrt{96}$ closer to?
How do you know?

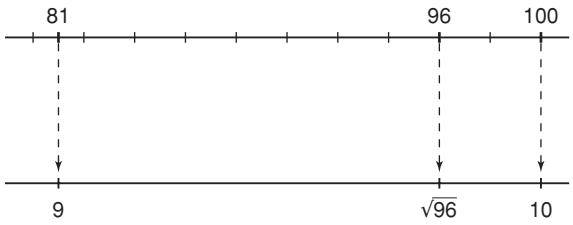
A Solution

$81 < 96 < 100$
So, $\sqrt{81} < \sqrt{96} < \sqrt{100}$
$9 < \sqrt{96} < 10$
$\sqrt{96}$ is between 9 and 10.
96 is closer to 100 than to 81.
So, $\sqrt{96}$ is closer to $\sqrt{100}$, or 10.

Example 1
Another Solution

Use number lines.

96 is between 81 and 100, but closer to 100.
So, $\sqrt{96}$ is between $\sqrt{81}$ and $\sqrt{100}$,
but closer to $\sqrt{100}$, or 10.



Example 2

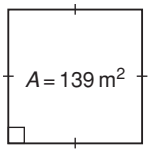
A square garden has area 139 m^2.

a) What are the approximate dimensions of the garden to two decimal places?

b) Net-wire fencing is needed to keep out coyotes. About how much fencing would be needed around the garden?

A Solution

a) Draw a square to represent the garden.



The side length of the square is: $\sqrt{139}$
Estimate:
$121 < 139 < 144$
So, $11 < \sqrt{139} < 12$
With a calculator, use guess and test to refine the estimate.
Try 11.5: $11.5 \times 11.5 = 132.25$ (too small)
Try 11.8: $11.8 \times 11.8 = 139.24$ (too large, but close)
Try 11.78: $11.78 \times 11.78 = 138.7684$ (close)
Try 11.79: $11.79 \times 11.79 = 139.0041$ (very close)

The side length of the garden is 11.79 m, to two decimal places.

b) To find how much fencing is needed, find the perimeter of the garden.
The perimeter of the garden is about:
$4 \times 11.79 \text{ m} = 47.16 \text{ m}$

To be sure there is enough fencing, round up.
About 48 m of fencing are needed to go around the garden.

Discuss the ideas

1. Which type of number has an exact square root?
2. Which type of number has an approximate square root?
3. How can you use perfect squares to estimate a square root, such as $\sqrt{8}$?

Practice

Check

4. Find.

a) $\sqrt{15 \times 15}$

b) $\sqrt{22 \times 22}$

c) $\sqrt{3 \times 3}$

d) $\sqrt{1 \times 1}$

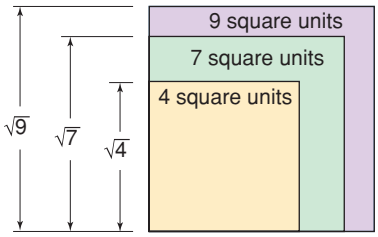
5. Between which two consecutive whole numbers is each square root?
How do you know?

a) $\sqrt{5}$ b) $\sqrt{11}$

c) $\sqrt{57}$ d) $\sqrt{38}$

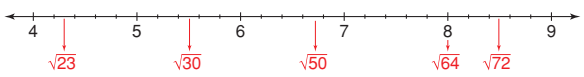
e) $\sqrt{171}$ f) $\sqrt{115}$

6. Copy this diagram on grid paper.
Then estimate the value of $\sqrt{7}$ to one decimal place.



7. Use the number line below.

a) Which placements are good estimates of the square roots?
Explain your reasoning.



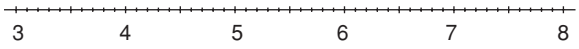
b) Use the number line to estimate the value of each square root that is incorrectly placed.

Apply

8. Use a copy of this number line.
Place each square root on the number line to show its approximate value.

a) $\sqrt{11}$ b) $\sqrt{40}$

c) $\sqrt{30}$ d) $\sqrt{55}$



9. In each pair, is the given whole number greater than, less than, or equal to the square root?
Justify your answer.

a) 7, $\sqrt{14}$

b) 8, $\sqrt{60}$

c) 11, $\sqrt{121}$

d) 12, $\sqrt{150}$

10. Which whole number is each square root closer to?
How do you know?

a) $\sqrt{58}$ b) $\sqrt{70}$

c) $\sqrt{90}$ d) $\sqrt{151}$

11. Is each statement true or false? Explain.

a) $\sqrt{17}$ is between 16 and 18.

b) $\sqrt{5} + \sqrt{5}$ is equal to $\sqrt{10}$.

c) $\sqrt{131}$ is between 11 and 12.

12. Use guess and test to estimate each square root to two decimal places. Record each trial.

a) $\sqrt{23}$ **b)** $\sqrt{13}$ **c)** $\sqrt{78}$

d) $\sqrt{135}$ **e)** $\sqrt{62}$ **f)** $\sqrt{45}$

13. Find the approximate side length of the square with each area. Give each answer to one decimal place.

a) 92 cm^2 **b)** 430 m^2

c) 150 cm^2 **d)** 29 m^2

14. Which estimates are good estimates of the square roots? Explain your reasoning.

a) $\sqrt{17}$ is about 8.50.

b) $\sqrt{20}$ is about 4.30.

c) $\sqrt{8}$ is about 2.83.

d) $\sqrt{34}$ is about 5.83.

15. **Assessment Focus** A student uses a square canvas for her painting. The canvas has area 5 m^2. She wants to frame her artwork.

a) What are the dimensions of the square frame to two decimal places?

b) The framing can be purchased in 5-m or 10-m lengths. Which length of framing should she purchase? Justify your choice.

16. A square lawn is to be reseeded. The lawn has area 152 m^2.

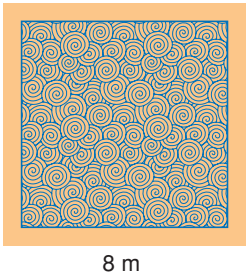
a) What are the approximate dimensions of the lawn to two decimal places?

b) A barrier of yellow tape is placed around the lawn to keep people off. About how much tape is needed?

17. Which is the closer estimate of $\sqrt{54}$: 7.34 or 7.35? How did you find out?

18. Most classrooms are rectangles. Measure the dimensions of your classroom. Calculate its area. Suppose your classroom was a square with the same area. What would its dimensions be?

19. **Take It Further** A square carpet covers 75% of the area of a floor. The floor is 8 m by 8 m.



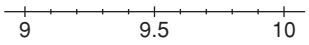
a) What are the dimensions of the carpet? Give your answer to two decimal places.

b) What area of the floor is not covered by the carpet?

20. **Take It Further** Is the product of two perfect squares always, sometimes, or never a perfect square? Investigate to find out. Write about your findings.

21. **Take It Further** An approximate square root of a whole number is 7.67. Is the whole number closer to 49 or 64? How do you know?

22. **Take It Further** Write five numbers whose square roots are between 9 and 10. Explain your strategy.



23. **Take It Further** Simplify each expression. Give your answer to two decimal places when necessary.

a) $\sqrt{81} + \sqrt{16}$

b) $\sqrt{81 + 16}$

c) $\sqrt{\sqrt{81} + 16}$

d) $\sqrt{81 + \sqrt{16}}$

e) $\sqrt{\sqrt{81} + \sqrt{16}}$

24. **a)** Estimate each square root to two decimal places.

i) $\sqrt{2}$ **ii)** $\sqrt{200}$ **iii)** $\sqrt{20\,000}$

b) Look at your results in part a. What patterns do you see?

c) Use the patterns in part b to estimate.

Reflect

What is your favourite method for estimating a square root of a number that is not a perfect square?
Use an example to explain how you would use your method.