

6.1

Solving Equations

Focus Solve equations by inspection and by systematic trial.

Look at the algebraic expressions and equations below.
Which ones are equations? Which ones are expressions?
How do you know?

$3n + 12$

$3n = 12$

$5x + 2$

$5x + 2 = 27$

Explore



On the way home from school,
10 students got off the bus at the first stop.
There were then 16 students on the bus.
How many students were on the bus
when it left the school?
How many different ways can you solve the problem?

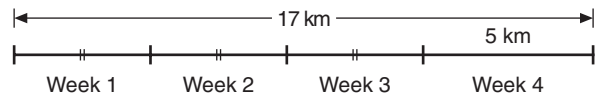


Reflect & Share

Discuss your strategies for finding the answer
with another pair of classmates.
Did you use an equation?
Did you use reasoning?
Did you draw a picture?
Justify your choice.

Connect

Janet walked a total of 17 km in February.
She walked the same number of kilometres
in each of the first 3 weeks.
Then she walked 5 km in the fourth week.
How many kilometres did Janet walk
in each of the first 3 weeks?



Let d represent the distance Janet walked, in kilometres, in each of the first 3 weeks.

So $3 \times d$, or $3d$, represents the total number of kilometres Janet walked in the first 3 weeks.

She walked 5 km in the fourth week, for a total of 17 km.

The equation is: $3d + 5 = 17$

When we use the equation to find the value of d , we *solve the equation*.

Here are 2 ways to solve this equation.

Method 1: By Systematic Trial

$$3d + 5 = 17$$

We choose a value for d and substitute.

$$\begin{aligned} \text{Try } d = 2. \quad 3d + 5 &= 3 \times 2 + 5 \\ &= 6 + 5 \\ &= 11 \end{aligned}$$

11 is too small, so choose a greater value for d .

$$\begin{aligned} \text{Try } d = 5. \quad 3d + 5 &= 3 \times 5 + 5 \\ &= 15 + 5 \\ &= 20 \end{aligned}$$

20 is too large, so choose a lesser value for d .

$$\begin{aligned} \text{Try } d = 4. \quad 3d + 5 &= 3 \times 4 + 5 \\ &= 12 + 5 \\ &= 17 \end{aligned}$$

This is correct.

Janet walked 4 km during each of the first 3 weeks of February.

Method 2: By Inspection

$$3d + 5 = 17$$

We first find a number which, when added to 5, gives 17.

$$3d + 5 = 17$$

We know that $12 + 5 = 17$.

So, $3d = 12$

Then we find a number which, when multiplied by 3, has product 12.

We know that $3 \times 4 = 12$; so $d = 4$

Janet walked 4 km during each of the first 3 weeks of February.

Writing and solving equations is a useful strategy for solving problems.

Systematic trial means choosing a value for the variable, then checking by substituting. Use the answer and reasoning to choose the next value to check.



By inspection means finding the value for the variable by using these types of number facts: addition, subtraction, multiplication, and division

We say that the value $d = 4$ makes the equation $3d + 5 = 17$ true.

Any other value of d , such as $d = 6$,

would not make the equation true.

The value $d = 4$ is the only solution to the equation.

That is, there is only one value of d that makes the equation true.

$3 \times 6 + 5$ does not equal 17.

Example

For each situation, write an equation.

Ben has a large collection of baseball caps.

- a) Ben takes y caps from a group of 18 caps.

There are 12 caps left.

How many caps did Ben take away?

Solve the equation by inspection.

- b) Ben put k caps in each of 6 piles.

There are 108 caps altogether.

How many caps did Ben put in each pile?

Solve the equation by systematic trial.

- c) Ben shares n caps equally among 9 piles.

There are 6 caps in each pile.

How many caps did Ben have?

Solve the equation by inspection.

- d) Ben combines p groups of 4 caps each into one large group.

He then takes away 7 caps. There are 49 caps left.

How many groups of 4 caps did Ben begin with?

Solve the equation by systematic trial.



A Solution

- a) 18 subtract y equals 12.

$$18 - y = 12$$

Which number subtracted from 18 gives 12?

We know that $18 - 6 = 12$; so $y = 6$.

Ben took away 6 caps.

- b) 6 times k equals 108.

$$6k = 108$$

Try $k = 15$.

$$\begin{aligned} 6k &= 6(15) \\ &= 90 \end{aligned}$$

90 is too small, so choose a greater value for k .

Recall: $6(15) = 6 \times 15$

Try $k = 20$. $6k = 6(20)$
 $= 120$

120 is too large, so choose a lesser value for k .

Try $k = 17$. $6k = 6(17)$
 $= 102$

102 is too small, but it is close to the value we want.

Try $k = 18$. $6k = 6(18)$
 $= 108$

This is correct.

Ben put 18 caps in each pile.

- c) n divided by 9 equals 6.

$$n \div 9 = 6, \text{ or } \frac{n}{9} = 6$$

Which number divided by 9 gives 6?

We know that $54 \div 9 = 6$; so $n = 54$.

Ben had 54 caps.

- d) 4 times p subtract 7 equals 49.

$$4p - 7 = 49$$

Since $4 \times 10 = 40$, we know we need to start with a value for p greater than 10.

$$\begin{aligned} \text{Try } p = 12. \quad 4p - 7 &= 4 \times 12 - 7 \\ &= 48 - 7 \\ &= 41, \text{ which is too small} \end{aligned}$$

41 is 8 less than 49, so we need two more groups of 4.

$$\begin{aligned} \text{Try } p = 14. \quad 4p - 7 &= 4 \times 14 - 7 \\ &= 56 - 7 \\ &= 49 \end{aligned}$$

This is correct.

Ben began with 14 groups of 4 caps each.

Practice

1. Look at the algebraic expressions and equations below.

Which are expressions? Equations?

How do you know?

a) $4w = 48$

b) $g - 11$

c) $3d + 5$

d) $\frac{x}{12} = 8$

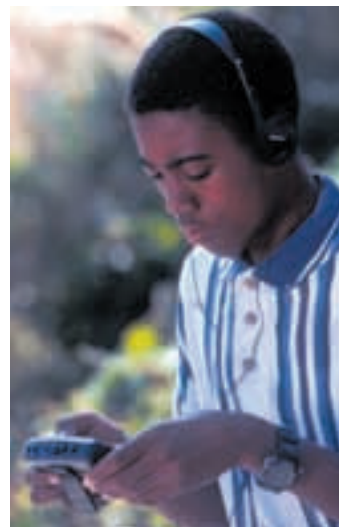
e) $\frac{j-5}{10}$

f) $6z + 1 = 67$

- 2.** Solve each equation in question 1 by inspection or by systematic trial.
Explain why you chose the method you did.

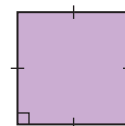
- 3.** Shenker gives 10 CDs to his brother.
Shenker then has 35 CDs.
- Write an equation you can solve to find how many CDs Shenker had to begin with.
 - Solve the equation.

- 4.** Write an equation for each sentence.
Solve each equation by inspection.
- Seven more than a number is 18.
 - Six less than a number is 24.
 - Five times a number is 45.
 - A number divided by six is 7.
 - Three more than four times a number is 19.

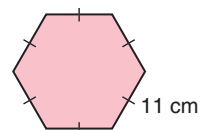


- 5.** Write an equation you could use to solve each problem.
Solve each equation by systematic trial.
- Aiko bought 14 DVDs for \$182.
She paid the same amount for each DVD.
How much did each DVD cost?
 - Kihew collects beaded leather bracelets. She lost 14 of her bracelets.
Kihew has 53 bracelets left.
How many bracelets did she have to begin with?
 - Manuel gets prize points for reading books.
He needs 100 points to win a set of tangrams.
Manuel has 56 points. When he reads 11 more books,
he will have 100 points.
How many points does Manuel get for each book he reads?

- 6.** The perimeter of a square is 48 cm.
- Write an equation you can solve to find the side length of the square.
 - Solve the equation.



- 7.** The side length of a regular hexagon is 11 cm.
- Write an equation you can solve to find the perimeter of the hexagon.
 - Solve the equation.



8. Use questions 6 and 7 as a guide.
- Write your own problem about side length and perimeter of a figure.
 - Write an equation you can use to solve the problem.
 - Solve the equation.
9. **Assessment Focus** Eli has 130 key chains. He keeps 10 key chains for himself, then shares the rest equally among his friends. Each friend then has 24 key chains.
- Write an equation you can solve to find how many friends were given key chains.
 - Solve the equation by inspection, then by systematic trial. Which method was easier to use? Explain your choice.
10. Find the value of n that makes each equation true.
- a) $3n = 27$ b) $2n + 3 = 27$ c) $2n - 3 = 27$ d) $\frac{n}{3} = 27$
11. **Take It Further** Write a problem that can be described by each equation. Solve each equation. Which equation was the most difficult to solve? Why do you think so?
- $2x - 1 = 5$
 - $4y = 24$
 - $\frac{z}{38} = 57$
 - $5x + 5 = 30$

Math Link

Dr. Edward Doolittle, a Mohawk Indian, was the first Indigenous person in Canada to obtain a PhD in Mathematics. Dr. Doolittle has taught at the First Nations University in Saskatchewan, and he is currently an Assistant Professor of Mathematics at the University of Regina. One of Dr. Doolittle's goals is to show his students how much fun mathematics can be. In addition to his academic interests, Dr. Doolittle also writes and performs comedy sketches for radio.



Reflect

How do you decide whether to solve an equation by inspection or by systematic trial?
 How might using a calculator affect your decision?
 Give examples to illustrate your thinking.