3.8 Solving Problems with Fractions

Focus Identify the operation required to solve a problem involving fractions.

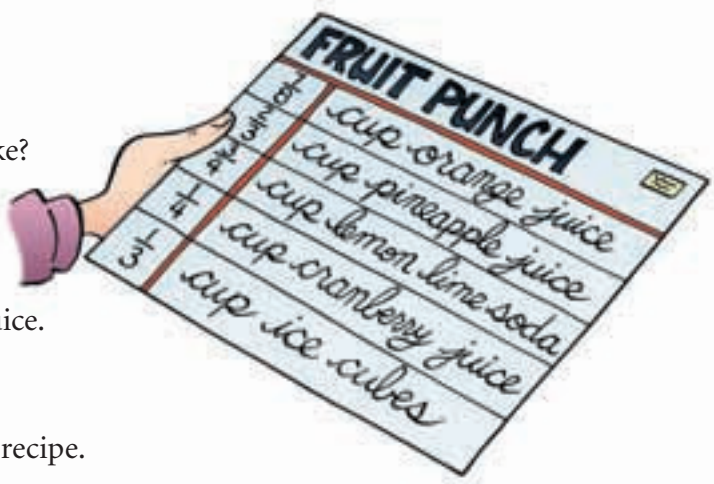
A Grade 8 class is going to a canoe competition.
There are 24 students in the class.
Each canoe holds 4 people.
How many canoes does the class need?
How did you know which operation to use?

Investigate

Work with a partner.

Aidan wants to make this fruit punch recipe.
Answer each of these questions.
Show your work.

- How many cups of punch does the recipe make?
- Suppose Aidan makes the punch, then pours himself $\frac{3}{4}$ cup of punch.
  How much punch does he have left?
- Suppose Aidan only has $\frac{1}{3}$ cup of pineapple juice.
  How much of each of the other ingredients does he need to keep the flavour the same?
- Suppose Aidan decides to make one-third the recipe.
  How much soda will he need?



Reflect & Share

Compare your answers and strategies with those of another pair of classmates.
Did you solve the problems the same way?
How did you decide which operation to use each time?

Connect

When solving word problems, it is important to identify the operation or operations needed to solve the problem.
To identify the operation:

- Think about the situation.
- Make sense of the problem by explaining it in your own words, drawing a picture, or using a model.
- Think about what is happening in the problem. Sometimes, key words can help you identify the operation to use.

For example, "total" suggests adding,
"less than" suggests subtracting,
"times" suggests multiplying, and
"shared" suggests dividing.

Although key words may help identify an operation, the operation must make sense in the context of the problem.

Example 1

Kassie worked on her science project for $\frac{3}{4}$ h on Tuesday and $\frac{5}{6}$ h on Wednesday. She spent Thursday finishing her math homework.

a) How long did Kassie work on her science project altogether?

b) How much longer did Kassie work on the project on Wednesday than on Tuesday?

c) Altogether, Kassie spent 2 h on school work over the 3 days. How long did Kassie spend on her math homework?

A Solution

a) The word "altogether" suggests addition.
Add: $\frac{3}{4} + \frac{5}{6}$

$$\frac{3}{4} + \frac{5}{6} = \frac{9}{12} + \frac{10}{12}$$
$$= \frac{19}{12}$$
$$= \frac{12}{12} + \frac{7}{12}$$
$$= 1\frac{7}{12}$$

Kassie worked on her science project for $1\frac{7}{12}$ h.

Write an equivalent fraction with denominator 12 for each fraction.

$\frac{3}{4} = \frac{9}{12}$ $\quad \frac{5}{6} = \frac{10}{12}$

b) The words "longer … than" suggest subtraction.
To find how much longer Kassie worked on Wednesday than on Tuesday, subtract: $\frac{5}{6} - \frac{3}{4}$

$$\frac{5}{6} - \frac{3}{4} = \frac{10}{12} - \frac{9}{12}$$
$$= \frac{1}{12}$$

Kassie worked $\frac{1}{12}$ h longer on Wednesday than on Tuesday.

c) The word "altogether" suggests addition.
However, to find the time Kassie spent doing math homework, we subtract.
From part a, we know Kassie spent $1\frac{7}{12}$ h on her science project.
Altogether, Kassie spent 2 h on school work over the 3 days.
So, the time spent on her math homework is:

$$2 - 1\frac{7}{12} = 1\frac{12}{12} - 1\frac{7}{12}$$
$$= \frac{5}{12}$$

Kassie spent $\frac{5}{12}$ h on her math homework.

Example 2

Dakota volunteered at a gift-wrapping booth for a local charity.

He volunteered for $2\frac{3}{4}$ h and wrapped 11 gift boxes.

His friend Winona volunteered $1\frac{1}{3}$ times as long.

a) How long did Dakota spend wrapping each gift box?
What assumptions do you make?

b) How many hours did Winona volunteer?

A Solution

a) Think what the problem means:
We are given a time for many and we have to find a time for one.
This suggests division.
$2\frac{3}{4} \div 11$
Write the mixed number as an improper fraction.
$2\frac{3}{4} = \frac{11}{4}$

So, $2\frac{3}{4} \div 11 = \frac{11}{4} \div 11$ Write the reciprocal of the divisor, then multiply.

$= \frac{11}{4} \times \frac{1}{11}$ Simplify. Divide by the common factor 11.

$= \frac{{}^{1}\cancel{11} \times 1}{4 \times \cancel{11}_{1}}$

$= \frac{1}{4}$

Dakota spent $\frac{1}{4}$ h wrapping each gift box.

It was assumed that Dakota wrapped for the entire time and that he spent the same amount of time wrapping each box.

b) The words "times as long" suggest multiplication.
So, Winona volunteered for $1\frac{1}{3}$ of the $2\frac{3}{4}$ h that Dakota volunteered.
$2\frac{3}{4} \times 1\frac{1}{3}$
Write the mixed numbers as improper fractions.
$2\frac{3}{4} = \frac{11}{4}$ and $1\frac{1}{3} = \frac{4}{3}$

So, $2\frac{3}{4} \times 1\frac{1}{3} = \frac{11}{4} \times \frac{4}{3}$

$= \frac{11 \times \cancel{4}^{1}}{\cancel{4}_{1} \times 3}$ Simplify. Divide by the common factor 4.

$= \frac{11}{3}$

$= 3\frac{2}{3}$

Winona volunteered for $3\frac{2}{3}$ h.

Discuss the ideas

1. What other words indicate addition, subtraction, multiplication, and division?
2. What other questions could you ask using the data in *Example 2*?
What operation would you use to answer each question?

Practice

Check

3. Which operation would you use to solve each problem? How can you tell?

a) Noel used $\frac{2}{3}$ cup of milk and $\frac{1}{4}$ cup of oil to make cookies. How much liquid did he use altogether?

b) One-third of the cars in the parking lot are silver. There are 165 cars in the lot. How many cars are silver?

c) Shania has $\frac{3}{8}$ cup of yogurt. She needs $\frac{3}{4}$ cup of yogurt to make a smoothie. How much more yogurt does she need?

d) Part of a pizza was shared equally between two friends. Each friend got $\frac{5}{12}$ of the whole pizza. How much pizza was shared?

Solve each problem. For each problem, explain how you decided which operation or operations to use.

4. Chad mixed $\frac{2}{3}$ of one can of yellow paint and $\frac{1}{4}$ of one can of white paint to paint a wall in his bedroom. How much paint did he have altogether?

5. Vivi scored 5 goals in the Saskatoon Sticks lacrosse tournament. This was $\frac{1}{8}$ of her team's goals. How many goals did Vivi's team score altogether?

Apply

6. Parent-teacher interviews were held on Thursday. Of those parents who attended, $\frac{1}{6}$ attended in the morning, $\frac{1}{3}$ attended in the afternoon, and the rest attended in the evening.

a) What fraction of the parents attended in the evening?

b) Thirty parents attended the interviews. How many parents attended in the evening?

7. Patti works in a coffee shop. She usually takes $\frac{3}{4}$ h for lunch. One day the shop was very busy and Patti's manager asked her to shorten her lunch break by $\frac{1}{6}$ h. What fraction of an hour did Patti take for lunch that day?

8. Katrina's monthly salary is $2400. She uses $\frac{2}{5}$ of this money for rent. How much rent does Katrina pay?

9. A snail travelled 48 cm in $\frac{2}{3}$ h. Suppose the snail moved at a constant speed and made no stops. How far would the snail travel in 1 h?

10. Beven has a collection of 72 music CDs. One-sixth of the CDs are dance music, $\frac{1}{4}$ are hip hop, and $\frac{3}{8}$ are reggae. The rest of the CDs are rock music. What fraction of her CDs are rock music?

11. **Assessment Focus** A jug contains $2\frac{1}{2}$ cups of juice. Shavon pours $\frac{3}{8}$ cup of juice into each of three glasses, then $\frac{5}{6}$ cup of juice into a fourth glass.

a) Estimate the fraction of the apple juice that remains in the jug.

b) Calculate the total amount of juice in the 3 glasses that contain the same amount.

c) Calculate the total amount of juice in the 4 glasses.

d) How much juice remains in the jug after the 4 glasses have been poured?

12. The Hendersons went on a driving vacation. They decided to travel $\frac{1}{3}$ of the distance on the first day. Owing to bad weather, they had to stop for the night having gone only $\frac{1}{4}$ of the distance to the one-third point. What fraction of the total distance did they travel the first day?

13. Howie works at a petting zoo. He fed a piglet $\frac{1}{5}$ of a bottle of milk, then gave $\frac{3}{4}$ of what was left to a calf. How much of the bottle of milk did the calf drink?

14. Nathan used $2\frac{5}{6}$ loaves of bread to make sandwiches for a lunch. He made equal numbers of 4 different types of sandwiches. What fraction of a loaf of bread did Nathan use for each type of sandwich?

15. **Take It Further** A steward reported to an airport official before takeoff that three-fifths of the passengers were women, three-eighths were men, and one-twentieth were children. After thinking for a moment, the official seemed puzzled and asked the steward to repeat the fractions. Why do you think the official was puzzled? Explain.

Reflect

Use the fractions $\frac{1}{2}$ and $\frac{7}{8}$. Write 4 problems. Each problem should require a different operation (addition, subtraction, multiplication and division). Solve each problem. How did you decide which operation to use?