11.4 Applications of Probability

Focus Solve probability problems.

Explore

Work with a partner.
You will play the *Sum and Product* game.
You will need 4 blank cards and a bag.
Write the numbers from 1 to 4 on the cards.
Place the cards in the bag.
Each person picks a card.
Both of you find the sum and the product of the two numbers.
One of you is Player A, the other is Player B.
If the sum is less than or equal to the product, Player A gets a point.
If the sum is greater than the product, Player B gets a point.

- Who is likely to win? Explain your reasoning.
- Play the game several times; you choose how many times. Who won?
- How does your prediction of the winner compare with your result?

Reflect & Share

Compare your results with those of another pair of classmates.
Work together to come up with an explanation of who is more likely to win.

Connect

- Probability can be expressed as a fraction, a decimal, or a percent. When probability is expressed as a percent, we use the word "chance."
  For example, the weather forecast is a 40% chance of rain today.
  This means that the probability of rain is: $\frac{40}{100} = 0.4$
- When an outcome is certain, the probability of it occurring is 1. For example, when we toss a coin, the probability of it landing heads or tails is 1.
  When an outcome is impossible, the probability of it occurring is 0. For example, when we roll a number cube labelled 1 to 6, the probability of a 7 showing is 0.

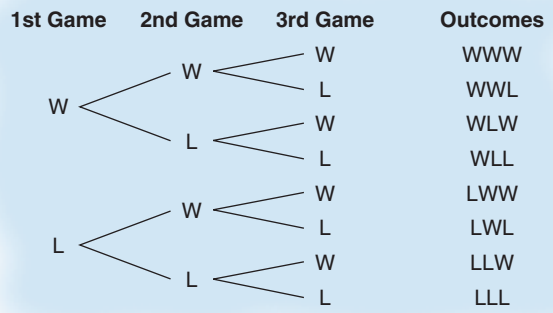
Example

Flick This is an Ultimate Frisbee team.
The team plays 3 games against 3 other teams.
All 4 teams have equal chances of winning.

a) What is the chance that Flick This will win all three of its games?
b) What is the chance that Flick This will win exactly one game?
c) What is the chance that Flick This will win at least two games?

Solution

For any game, the possible outcomes are win (W) or lose (L).
These outcomes are equally likely. Draw a tree diagram to list the possible results of 3 games for Flick This.



There are 8 possible outcomes.

a) There is 1 outcome in which Flick This wins all three games: WWW
So, the probability of 3 wins is: $\frac{1}{8} = 0.125$
So, the chance of winning 3 games is 12.5%.

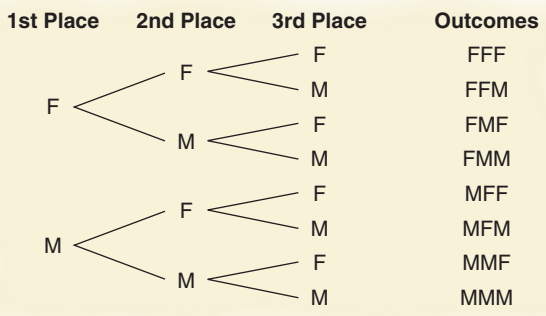
The word "exactly" is included because "winning one game" might be interpreted as winning one or more games.

b) There are 3 outcomes in which Flick This wins exactly one game: WLL, LWL, LLW
So, the probability of winning exactly 1 game is: $\frac{3}{8} = 0.375$
So, the chance of winning exactly 1 game is 37.5%.

c) There are 4 outcomes in which Flick This wins at least two games: WWW, WWL, WLW, LWW
So, the probability of winning at least 2 games is: $\frac{4}{8} = \frac{1}{2} = 0.5$
So, the chance of winning at least 2 games is 50%.

Practice

1. The 1st, 2nd, and 3rd place winners of a contest can be female or male. This tree diagram shows the possible outcomes of the contest.



a) How many possible outcomes are there?
b) What is the probability that all the winners are female?
c) What is the probability that none of the winners is male?
d) How are the answers to parts b and c related? Explain.

2. On this spinner, the pointer is spun once.
The colour is recorded.
The pointer is spun a second time.
The colour is recorded.

a) Suppose you win if you spin the same colour on both spins.
What are your chances of winning?
b) Suppose you win if you spin two different colours.
What are your chances of winning?

3. **a)** Three coins are tossed. Find the chance of tossing:
i) one heads and two tails **ii)** exactly two heads
iii) at least two tails **iv)** no heads
b) Why do we need the words "at least" in part a, iii?
What if these words were left out?
How would the answer change?
c) Why do we need the word "exactly" in part a, ii? What if this word was left out? How would the answer change?

4. At a carnival, the game with the least chance of winning often has the greatest prize. Explain why this might be.

5. There are four children in a family.
What is the chance of each event?

a) There are two boys and two girls.

b) There is at least one girl.

c) All four children are of the same gender.

6. **Assessment Focus** The school cafeteria has this lunch menu.
A student chooses a sandwich and a vegetable. Assume the choice is random.

a) Find the probability of each possible combination.

b) Suppose 3 desserts were added to the menu. Each student chooses a sandwich, a vegetable, and a dessert.
How would the probabilities of possible combinations change?
Use examples to explain your thinking.

Lunch Menu	
Sandwich	**Vegetable**
Grilled Cheese	Broccoli
Chicken	Carrots
Tuna	

Take It Further

7. At the school carnival, there is a game with two spinners.

Spinner A

Spinner B

You get two spins.
You may spin the pointer on each spinner once, or spin the pointer on one spinner twice.
If you get pink on one spin and yellow on another spin (the order does not matter), you win.
To have the greatest chance of winning, what should you do?
Explain.

Calculator Skills

Which three consecutive prime numbers have a product of 7429 and a sum of 59?

Reflect

How is probability related to chance?
Use an example in your explanation.

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1.a) 8 **b)** 0.125 **c)** 0.125

d) They are the same; if all the winners are female, none of the winners can be male, which is the answer to part c.

2.a) 25% **b)** 75%

3.a) **i)** 37.5% **ii)** 37.5% **iii)** 50% **iv)** 12.5%

b) For example: "At least" includes 2 tails and 3 tails. If these words were left out, 3 tails would not be included, and the probability of getting tails would be lower.

c) For example: "Exactly" means 3 heads are not included. If this word was left out, this could mean 2 heads or 3 heads, and the probability would change.

4. For example: People are not likely to win, so the carnival owners can afford to be generous with the prize.

5.a) 37.5% **b)** 93.75% **c)** 12.5%

6.a) $\frac{1}{6}$

b) For example: There would be 18 different combinations. Each combination would have a probability of $\frac{1}{18}$.

7. For example: Best combination is Spinner A for pink, then Spinner B for yellow, or Spinner B for yellow, then Spinner A for pink.

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1. $6.97

2. For example: It depends on the number of hours worked. If he works less than 22 h, $96 per week is a better deal. If he works 22 h or more, $4.50/h is better.

3.a) 81 cm^2 **b)** 216 cm^3

4. For example: $\frac{1}{32}$

6. a and b are the same; the final price is $73.59 in both cases.

7. $18

9. 136

10.a) 6 **b)** 0.25

11.a) 0.125 **b)** About 0.396

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1.a) Banana, carrots, yogurt; banana, carrots, cheese; banana, celery, yogurt; banana, celery, cheese; banana, cucumber, yogurt; banana, cucumber, cheese; orange, carrots, yogurt; orange, carrots, cheese; orange, celery, yogurt; orange, celery, cheese; orange, cucumber, yogurt; orange, cucumber, cheese; apple, carrots, yogurt; apple, carrots, cheese; apple, celery, yogurt; apple, celery, cheese; apple, cucumber, yogurt; apple, cucumber, cheese

b) 3 **c)** 14 **d)** 12

2.a) HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, HTTH, THHT, THTH, TTHH, HTTT, THTT, TTHT, TTTH, TTTT

b) 4 **c)** 6 **d)** 5

3.a) 0.9 **b)** 13 500

4.a) 0.487 **b)** 0.513 **c)** 0.715

d) 0.167 **e)** 0.395

5.a) False; it is very unlikely, but it could happen.

b) True; it is very unlikely.

c) False; this outcome will not always occur in practice.

d) True; with a greater number of trials, the experimental and theoretical probabilities are likely to be close.

6.a) Yes; the circle is divided into 10 equal parts.

b) 0.1 **c)** 0.4 **d)** 0.5

7.a) 0.5 **b)** 0.25 **c)** 0.8 **d)** 0.45

8.a) 7 times **b)** 33 times **c)** 59 times **d)** 46 times

9.a) **i)** 25% **ii)** 0%

b) **i)** 37.5% **ii)** 87.5%

10. About 33%

Unit 11 Practice Test, page 429

1. Saturday, matinee, adult; Saturday, matinee, child; Saturday, matinee, senior; Saturday, evening, adult; Saturday, evening, child; Saturday, evening, senior; Sunday, matinee, adult; Sunday, matinee, child; Sunday, matinee, senior; Sunday, evening, adult; Sunday, evening, child; Sunday, evening, senior

2.a) 10 times; it is likely to appear once every 6 rolls.

b) 30 times; one-half the numbers are even.

c) 30 times; one-half the numbers are greater than 3.

d) 0 times; 9 is not a number on the cube.

3.a) 0.3 **b)** 0.353

c) For example: 27, if her batting average stays at 0.3.

4. Equally likely; for example: The probabilities are the same.

5. About 8 times

Cumulative Review, Units 1–11, page 434

1.a) 6, 12, 18, 24, 30 **b)** 9, 18, 27, 36, 45

c) 12, 24, 36, 48, 60

2.a) 1, 2, 3, 4, 6, 9, 12, 18, 36

b) 1, 3, 19, 57 **c)** 1, 3, 5, 15, 25, 75

3. Quilt B

4.a) $SA = 6c^2$ **b)** $SA = 2lw + 2lh + 2wh$

c) For example: Substitute c for l, w, and h.

5.a) $1\frac{5}{12}$ **b)** $5\frac{14}{15}$ **c)** $1\frac{7}{10}$

d) $1\frac{4}{5}$ **e)** $\frac{45}{8}$ or $5\frac{5}{8}$ **f)** $\frac{26}{10}$ or $2\frac{3}{5}$