3.7 Dividing Mixed Numbers

Focus Apply knowledge of dividing fractions to divide mixed numbers.

What strategies can you use to divide $\frac{5}{6} \div \frac{2}{3}$?

Investigate

Work with a partner.
Jeffrey wants to share some round Belgian waffles with his friends.
Suppose Jeffrey makes each portion $\frac{3}{4}$ of a waffle.

How many portions will he get from $4\frac{1}{2}$ waffles?
How can you find out?
Show your work.
Use models or diagrams to justify your strategy.

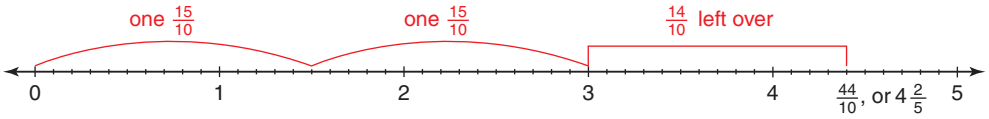
Reflect & Share

Compare your strategy with that of another pair of classmates.
Do you think your strategy will work with all mixed numbers?
Test it with $1\frac{1}{2} \div \frac{5}{8}$.

Connect

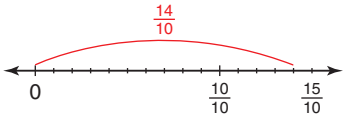
Here are three ways to divide mixed numbers.
In each method, the mixed numbers are first written as improper fractions.

➤ Use a number line.
To divide: $4\frac{2}{5} \div 1\frac{1}{2}$
$4\frac{2}{5} = \frac{22}{5}$ and $1\frac{1}{2} = \frac{3}{2}$
Write each fraction with a common denominator.
Since 2 and 5 have no common factors, a common denominator is $2 \times 5 = 10$.
$\frac{22}{5} = \frac{44}{10}$ and $\frac{3}{2} = \frac{15}{10}$
So, $4\frac{2}{5} \div 1\frac{1}{2} = \frac{44}{10} \div \frac{15}{10}$
This means: How many 15 tenths are in 44 tenths?
Use a number line divided in tenths.



From the number line, there are 2 groups of 15 tenths, with remainder 14 tenths.

Think: What fraction of 15 tenths is 14 tenths?



From the number line, 14 tenths is $\frac{14}{15}$ of 15 tenths.
So, $4\frac{2}{5} \div 1\frac{1}{2} = 2\frac{14}{15}$

➤ Use common denominators.
Divide: $4\frac{2}{5} \div 1\frac{1}{2}$
$4\frac{2}{5} = \frac{22}{5}$ and $1\frac{1}{2} = \frac{3}{2}$
Write each fraction with a common denominator.
$\frac{22}{5} = \frac{44}{10}$ and $\frac{3}{2} = \frac{15}{10}$
So, $4\frac{2}{5} \div 1\frac{1}{2} = \frac{44}{10} \div \frac{15}{10}$
$= 44 \div 15$
$= \frac{44}{15}$, or $2\frac{14}{15}$

Since the denominators are the same, divide the numerators.

➤ Use multiplication.

$4\frac{2}{5} \div 1\frac{1}{2} = \frac{22}{5} \div \frac{3}{2}$

Recall that dividing by $\frac{3}{2}$ is the same as multiplying by $\frac{2}{3}$.

$\frac{2}{3}$ is the reciprocal of $\frac{3}{2}$.

So, $\frac{22}{5} \div \frac{3}{2} = \frac{22}{5} \times \frac{2}{3}$

$= \frac{22 \times 2}{5 \times 3}$

$= \frac{44}{15}$

$= 2\frac{14}{15}$

Example 1

Divide. Estimate to check the quotient is reasonable.

$1\frac{7}{8} \div 1\frac{1}{4}$

A Solution

$1\frac{7}{8} \div 1\frac{1}{4}$

Change the mixed numbers to improper fractions.

$1\frac{7}{8} = \frac{15}{8}$ and $1\frac{1}{4} = \frac{5}{4}$

So, $1\frac{7}{8} \div 1\frac{1}{4} = \frac{15}{8} \div \frac{5}{4}$

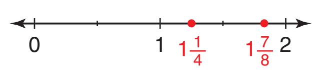
$= \frac{15}{8} \times \frac{4}{5}$

$= \frac{{}^{3}\cancel{15}}{{}_{2}\cancel{8}} \times \frac{\cancel{4}^{1}}{\cancel{5}_{1}}$

$= \frac{3 \times 1}{2 \times 1}$

$= \frac{3}{2}$, or $1\frac{1}{2}$

Estimate to check.



$1\frac{7}{8}$ is close to 2. $1\frac{1}{4}$ is close to 1.

So, $1\frac{7}{8} \div 1\frac{1}{4}$ is about $2 \div 1 = 2$.

Since $1\frac{7}{8}$ is less than 2, and $1\frac{1}{4}$ is greater than 1, the quotient will be less than 2.

Since $1\frac{1}{2}$ is close to 2, the quotient is reasonable.

Example 2

Brittany has a summer job in a bakery.
One day, she used $3\frac{3}{4}$ cups of chocolate chips to make chocolate-chip muffins.

A dozen muffins requires $\frac{3}{4}$ cup chocolate chips.

How many dozen chocolate-chip muffins did Brittany make that day?

A Solution

$3\frac{3}{4} \div \frac{3}{4}$

Change the mixed number to an improper fraction.

$3\frac{3}{4} = \frac{15}{4}$

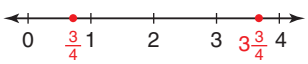
So, $3\frac{3}{4} \div \frac{3}{4} = \frac{15}{4} \div \frac{3}{4}$ Since the denominators are the same, divide the numerators.

$= 15 \div 3$

$= \frac{15}{3}$

$= 5$

Estimate to check.
$3\frac{3}{4}$ is close to 4. $\frac{3}{4}$ is close to 1.



So, $3\frac{3}{4} \div \frac{3}{4}$ is about $4 \div 1 = 4$.

Since 5 is close to 4, the solution is reasonable.
Brittany made 5 dozen chocolate-chip muffins.

Discuss the ideas

1. How is dividing mixed numbers similar to dividing fractions?
2. You have seen 3 methods for dividing mixed numbers. Which method are you likely to use most often? Justify your choice.
3. Why do we often write the quotient as a mixed number when we divide with mixed numbers?

Practice

Check

4. Write each mixed number as an improper fraction.

a) $4\frac{3}{8}$ **b)** $3\frac{2}{7}$ **c)** $6\frac{1}{6}$ **d)** $2\frac{1}{4}$

e) $1\frac{7}{10}$ **f)** $7\frac{2}{3}$ **g)** $2\frac{5}{9}$ **h)** $5\frac{2}{5}$

5. Write each improper fraction as a mixed number.

a) $\frac{14}{9}$ **b)** $\frac{16}{7}$ **c)** $\frac{24}{5}$ **d)** $\frac{21}{10}$

e) $\frac{15}{6}$ **f)** $\frac{23}{7}$ **g)** $\frac{17}{3}$ **h)** $\frac{25}{12}$

6. Use estimation. Which number is each quotient closer to?

a) $6\frac{1}{8} \div 2\frac{3}{4}$ 2 or 3

b) $7\frac{4}{5} \div 1\frac{3}{4}$ 3 or 4

c) $3\frac{1}{8} \div 2\frac{3}{4}$ 1 or 2

d) $9\frac{4}{7} \div 2\frac{1}{9}$ 4 or 5

7. Divide: $1\frac{4}{5} \div 2\frac{7}{10}$

a) Estimate the quotient.

b) Write each mixed number as an improper fraction.

c) Divide the improper fractions. Simplify first.

d) Is the quotient reasonable? How do you know?

Apply

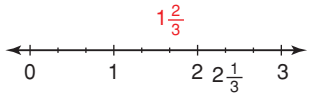
8. Use common denominators to find each quotient. Estimate to check the quotient is reasonable.

a) $3\frac{3}{4} \div 1\frac{1}{8}$ **b)** $1\frac{1}{6} \div 4\frac{1}{3}$

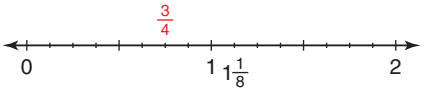
c) $3\frac{1}{4} \div 3\frac{1}{4}$ **d)** $\frac{2}{3} \div 1\frac{1}{9}$

9. Use a copy of each number line to illustrate each quotient.

a) $2\frac{1}{3} \div 1\frac{2}{3}$



b) $1\frac{1}{8} \div \frac{3}{4}$



10. Use multiplication to find each quotient. Estimate to check the quotient is reasonable.

a) $3\frac{2}{3} \div 5\frac{1}{4}$

b) $4\frac{3}{8} \div 1\frac{5}{16}$

c) $1\frac{3}{10} \div 3\frac{3}{5}$

d) $3\frac{2}{3} \div 3\frac{2}{3}$

11. Divide. Estimate to check the quotient is reasonable.

a) $1\frac{9}{10} \div 2\frac{2}{3}$ **b)** $2\frac{3}{4} \div 2\frac{1}{3}$

c) $1\frac{4}{5} \div 3\frac{1}{2}$ **d)** $1\frac{3}{8} \div 1\frac{3}{8}$

12. Maxine took $12\frac{1}{2}$ h to build a model airplane. She worked for $1\frac{1}{4}$ h each evening. How many evenings did Maxine take to complete the model?

13. Glenn ran $3\frac{1}{3}$ laps in $11\frac{2}{3}$ min. Assume Glenn took the same amount of time to complete each lap. How long did Glenn take to run one lap?

14. Assessment Focus Amelia has her own landscaping business. She ordered $10\frac{5}{8}$ loads of topsoil to fill large concrete planters. Each planter holds $1\frac{1}{2}$ loads of topsoil.

a) Estimate the number of planters that Amelia can fill.

b) Sketch a number line to illustrate the answer.

c) Calculate the number of planters that Amelia can fill.

d) What does the fraction part of the answer represent?

15. Write a story problem that could be solved using the expression $4\frac{2}{3} \div \frac{3}{5}$. Find the quotient to solve the problem. Estimate to check the solution is reasonable.

16. Use estimation. Which expression below has the greatest quotient? The least quotient? How do you know?

a) $\frac{8}{5} \div \frac{4}{3}$

b) $2\frac{3}{4} \div 1\frac{7}{8}$

c) $4\frac{8}{9} \div 2\frac{1}{8}$

d) $2\frac{1}{10} \div 1\frac{5}{6}$

17. Take It Further

a) Which of these quotients is a mixed number? How can you tell without dividing?

i) $4\frac{3}{8} \div 3\frac{2}{5}$

ii) $3\frac{2}{5} \div 4\frac{3}{8}$

b) Find the quotients in part a. What can you say about the order in which you divide mixed numbers?

18. Take It Further Which expression below has the greatest value? Give reasons for your answer. How could you find out without calculating each answer?

a) $3\frac{1}{5} \times \frac{1}{2}$ **b)** $3\frac{1}{5} \times \frac{2}{3}$

c) $3\frac{1}{5} \div \frac{2}{3}$ **d)** $3\frac{1}{5} \div \frac{2}{1}$

e) $3\frac{1}{5} + \frac{2}{3}$ **f)** $3\frac{1}{5} + \frac{3}{2}$

19. Take It Further One way to divide fractions is to use multiplication.

a) How could you multiply fractions by using division? Explain.

b) Do you think you would want to multiply fractions by using division? Why or why not?

Reflect

Suppose you divide one mixed number by another.
How can you tell, before you divide, if the quotient will be:

- greater than 1?
- less than 1?
- equal to 1?

Use examples in your explanation.