7.4 Solving Problems Involving Independent Events

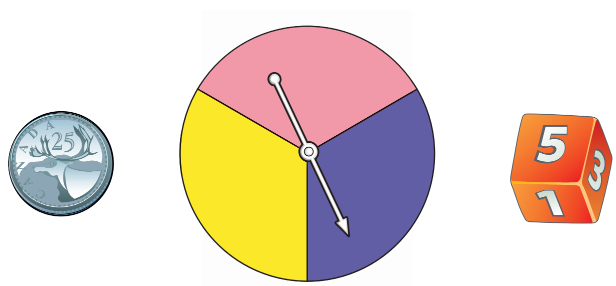
Focus Solve a problem that involves finding the probability of independent events.

In Lesson 7.3, you learned that when A and B are independent events, the probability of both A and B happening is the product of the probability of event A and the probability of event B.
P(A and B) = P(A) × P(B)

Investigate

Work with a partner.

➤ A probability experiment involves tossing a coin, rolling a die labelled 1 to 6, and spinning the pointer on a spinner with 3 congruent sectors coloured pink, purple, and yellow.
Use a tree diagram or a table to find the probability of each event.
- tossing heads, rolling a 2, and landing on purple
- tossing tails, rolling an even number, and landing on yellow
- tossing heads, rolling a 1 or 2, and landing on pink



➤ Predict a rule to find the probability of three independent events.
Use your rule to verify the probabilities you found above.

Reflect & Share

Compare your results and your rule with those of another pair of classmates.
Use your rule to find the probability of getting heads on 3 consecutive tosses of a coin.

Connect

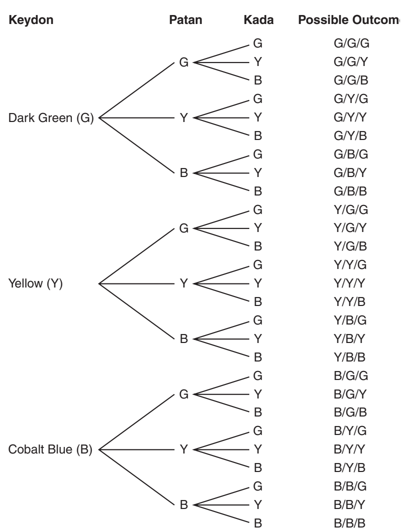
The rule for the probability of two independent events can be extended to three or more independent events.

Example 1

The students in a Grade 8 class were making buffalo horn beaded chokers.
Students could choose from dark green, yellow, and cobalt blue Crow beads.
Each student has the same number of beads of each colour.
Keydon, Patan, and Kada take their first beads without looking.
Find the probability that Keydon takes a yellow bead,
Patan takes a dark green bead, and Kada takes a yellow bead.

A Solution

Since each student has her own set of beads, the events are independent.
Use a tree diagram.



There are 27 possible outcomes. One outcome is Y/G/Y.
So the probability that Keydon takes a yellow bead,
Patan takes a dark green bead, and Kada takes a yellow bead is $\frac{1}{27}$.

In *Example 1*, the probability that Keydon takes a yellow bead is $P(Y) = \frac{1}{3}$.
The probability that Patan takes a dark green bead is $P(G) = \frac{1}{3}$.
The probability that Kada takes a yellow bead is $P(Y) = \frac{1}{3}$.
Note that: $\frac{1}{27} = \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$
Suppose the probability of Event A is P(A), the probability of Event B is P(B), and the probability of Event C is P(C).
Then, the probability that all A, B, and C occur is P(A and B and C).
If A, B, and C are independent events,
then $P(A \text{ and } B \text{ and } C) = P(A) \times P(B) \times P(C)$.

Example 2

On a particular day in July, there is a 20% probability of rain in Vancouver, a 65% probability of rain in Calgary, and a 75% probability of rain in Saskatoon.
What is the probability that it will rain in all 3 cities on that day?

A Solution

The events are independent.
Write each percent as a decimal, then multiply the decimals.
P(Rain in Vancouver) = 20%, or 0.20
P(Rain in Calgary) = 65%, or 0.65
P(Rain in Saskatoon) = 75%, or 0.75

$$\begin{aligned}\text{So, } P(\text{Rain V and C and S}) &= P(\text{Rain V}) \times P(\text{Rain C}) \times P(\text{Rain S}) \\ &= 0.20 \times 0.65 \times 0.75 \\ &= 0.0975, \text{ or } 9.75\%\end{aligned}$$

Use a calculator.

The probability that it will rain in all 3 cities on that day is 9.75%.

Example 2
Another Solution

Assume the events are independent.
P(Rain in Vancouver) = 20%, or $\frac{20}{100} = \frac{1}{5}$
P(Rain in Calgary) = 65%, or $\frac{65}{100} = \frac{13}{20}$
P(Rain in Saskatoon) = 75%, or $\frac{75}{100} = \frac{3}{4}$

$$\begin{aligned}\text{So, } P(\text{Rain V and C and S}) &= P(\text{Rain V}) \times P(\text{Rain C}) \times P(\text{Rain S}) \\ &= \frac{1}{5} \times \frac{13}{20} \times \frac{3}{4} \\ &= \frac{39}{400}\end{aligned}$$

The probability that it will rain in all 3 cities on that day is $\frac{39}{400}$.

Note that $\frac{39}{400} = 0.0975$.

Discuss the ideas

1. How can the rule for the probability of 2 independent events be extended to 4 or 5 independent events?
2. In *Example 2*, how can you use the answer to find the probability of it not raining in all 3 cities on that day?
3. Why are the events in *Example 2* considered independent?

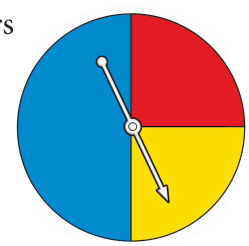
Practice

Check

4. One coin is tossed 3 times. Find the probability of each event:
   a) 3 heads
   b) 3 tails
   c) tails, then heads, then tails
   Use a tree diagram to verify your answers.

5. A red die, a blue die, and a green die are rolled. Each die is labelled 1 to 6. Find the probability of each event:
   a) a 2 on the red die, a 3 on the blue die, and a 4 on the green die
   b) a 4 on the red die, an even number on the blue die, and a number less than 3 on the green die

6. A spinner has 3 sectors coloured red, blue, and yellow. The pointer on the spinner is spun 3 times. Find the probability of each event:



   a) red, blue, and yellow
   b) blue, blue, and not red
   c) blue, blue, and blue

Apply

7. Stanley's bicycle lock has 4 dials, each with digits from 0 to 9. What is the probability that someone could guess his combination on the first try by randomly selecting a number from 0 to 9 four times?

8. A coffee shop has a contest. When you "lift the lid," you might win a prize. The probability of winning a prize is $\frac{1}{10}$. Suppose your teacher buys one coffee each day. Find the probability of each event:
   a) Your teacher will win a prize on each of the first 3 days of the contest.
   b) Your teacher will win a prize on the third day of the contest.
   c) Your teacher will not win a prize in the first 4 days of the contest.

9. **Assessment Focus** Nadine, Joshua, and Shirley each have a standard deck of playing cards. Each student randomly draws a card from the deck. Find the probability of each event:

a) Each student draws a heart.

b) Nadine draws a spade, Joshua draws a spade, and Shirley draws a red card.

c) Nadine does not draw a heart, Joshua draws a black card, and Shirley draws an ace. Show your work.

10. Preet writes a multiple-choice test. The test has 5 questions. Each question has 4 possible answers. Preet guesses each answer. Find the probability of each event:

a) She answers all 5 questions correctly.

b) She answers only the first 3 questions correctly.

c) She answers all the questions incorrectly.

11. Rocco chooses a 3-letter password for his e-mail account. He can use a letter more than once. What is the probability that someone else can access his e-mail by randomly choosing 3 letters?

12. Vanessa has 16 songs on a Classic Rock CD. Six of the songs are by the Beatles, 4 are by the Rolling Stones, 4 are by the Who, and 2 are by the Doors. Vanessa plays the CD. She selects a setting that randomly chooses songs to play. Find the probability of each event:

a) The first 3 songs played are by the Beatles.

b) The first 2 songs played are by the Rolling Stones and the next 2 songs are by the Beatles.

c) The first 2 songs played are by the Doors, and the next song played is either by the Beatles or the Rolling Stones.

13. A bag contains 5 blue marbles and 1 white marble. Susan draws a marble from the bag without looking, then replaces it in the bag. This is done 5 times.

a) What is the probability that the white marble is drawn 5 times in a row? Express your answer as a percent.

b) Suppose the white marble is drawn 5 times in a row. What is the probability the white marble will be picked on the next draw? Explain.

c) Is your answer to part b the same as the probability of drawing the white marble 6 times in a row? Why or why not?

14. Pancho wants to buy his teacher some flowers. The flower shop has 3 vases of cut flowers. One vase contains roses: 1 red, 4 yellow, and 3 white. A second vase contains carnations: 5 pink and 1 red. A third vase contains daisies: 1 yellow and 3 white. Pancho cannot decide so he closes his eyes and picks one flower from each vase.
Find the probability of each event.

a) Pancho picks a red rose, a pink carnation, and a white daisy.
b) Pancho picks a yellow or white rose, a red carnation, and a yellow daisy.
c) Pancho picks a red rose, a red carnation, and a red daisy.

15. **Take It Further** In gym class, students take turns shooting a basketball at the net. Each successful shot is worth 1 point. A player is awarded a second shot only if the first shot is successful. The player can score 0, 1, or 2 points in this situation. Suppose a player shoots with 70% accuracy.
Find the probability of each event.

a) She scores 0 points.
b) She scores 1 point.
c) She scores 2 points.

16. **Take It Further** A regular 6-sided die is rolled three times.

a) What is the probability of rolling three 6s in a row?
b) What is the probability of *not* rolling three 6s in a row?
c) Find the sum of your answers in parts a and b. Explain the result.

Reflect

Give examples of random independent events outside the classroom.
Why do you think the events are independent?

Math Link

Your World

The Coquihalla Highway is the only toll road in British Columbia. Those who run toll roads use probability to model the arrival of cars and trucks at the toll booths. This allows decisions to be made about how many toll booths should be open and how many toll booth operators are needed at any time of the day.