4.2 Circumference of a Circle

Focus Investigate the relationship between the circumference and diameter of a circle.

Explore

You will need 3 circular objects of different sizes, string, and a ruler.

➤ Each of you chooses one of the objects.
Use string to measure the distance around it.
Measure the radius and diameter of the object.
Record these measures.

➤ Repeat the activity until each of you has measured all 3 objects.
Compare your results.
If your measures are the same, record them in a table.
If your measures for any object are different, measure again to check.
When you agree upon the measures, record them in the table.

Object	Distance Around (cm)	Radius (cm)	Diameter (cm)
Can			

➤ What patterns do you see in the table?
How is the diameter related to the distance around?
How is the radius related to the distance around?

➤ For each object, calculate:
- distance around ÷ diameter
- distance around ÷ radius

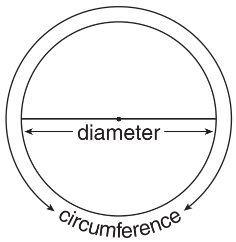
What do you notice?
Does the size of the circle affect your answers? Explain.

Reflect & Share

Compare your results with those of another group.
Suppose you know the distance around a circle.
How can you find its diameter?

Connect

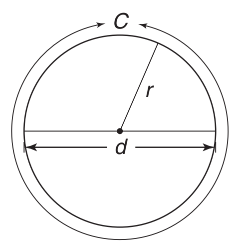
The distance around a circle is its **circumference**.
For any circle, the circumference, C, divided by the diameter, d, is approximately 3.
Circumference ÷ diameter ≐ 3, or $\frac{C}{d} \doteq 3$



The circumference of a circle is also the perimeter of the circle.

For any circle, the ratio $\frac{C}{d} = \pi$
The symbol π is a Greek letter that we read as "pi."
$\pi = 3.141\ 592\ 653\ 589\ldots$, or $\pi \doteq 3.14$

π is a decimal that never repeats and never terminates.
π cannot be written as a fraction.
For this reason, we call π an **irrational number**.



So, the circumference is π multiplied by d.
We write: $C = \pi d$

Since the diameter is twice the radius,
the circumference is also π multiplied by $2r$.
We write: $C = \pi \times 2r$, or $C = 2\pi r$

When we know the radius or diameter of a circle, we can use one of the formulas above to find the circumference of the circle.

The face of a toonie has radius 1.4 cm.

- To find the diameter of the face:
  The diameter $d = 2r$, where r is the radius
  Substitute: $r = 1.4$
  $d = 2 \times 1.4$
  $= 2.8$
  The diameter is 2.8 cm.

The circumference is a length, so its units are units of length such as centimetres, metres, or millimetres.

- To find the circumference of the face:

$C = \pi d$	OR	$C = 2\pi r$
Substitute: $d = 2.8$		Substitute: $r = 1.4$
$C = \pi \times 2.8$		$C = 2 \times \pi \times 1.4$
$\doteq 8.796$		$\doteq 8.796$
$\doteq 8.8$		$\doteq 8.8$

  The circumference is 8.8 cm, to one decimal place.

Use the π key on your calculator. If the calculator does not have a π key, use 3.14 instead.

- We can estimate to check if the answer is reasonable.
  The circumference is approximately 3 times the diameter:
  $3 \times 2.8 \text{ cm} \doteq 3 \times 3 \text{ cm}$
  $= 9 \text{ cm}$
  The circumference is approximately 9 cm.
  The calculated answer is 8.8 cm, so this answer is reasonable.

When we know the circumference, we can use a formula to find the diameter.
Use the formula $C = \pi d$.
To isolate d, divide each side by π.

$$\frac{C}{\pi} = \frac{\pi d}{\pi}$$

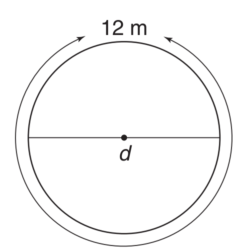
$$\frac{C}{\pi} = d$$

So, $d = \frac{C}{\pi}$

Example

An above-ground circular swimming pool has circumference 12 m.
Calculate the diameter and radius of the pool.
Give the answers to two decimal places.
Estimate to check the answers are reasonable.

A Solution



The diameter is: $d = \frac{C}{\pi}$
Substitute: $C = 12$
$d = \frac{12}{\pi}$
$= 3.8197\ldots$

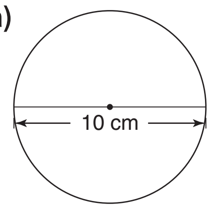
Use a calculator.
Do not clear your calculator.

The radius is $\frac{1}{2}$ the diameter, or $r = d \div 2$.
Divide the number in the calculator display by 2.
$r \doteq 1.9099$
The diameter is 3.82 m to two decimal places.
The radius is 1.91 m to two decimal places.

Since the circumference is approximately 3 times the diameter, the diameter is about $\frac{1}{3}$ the circumference.
One-third of 12 m is 4 m. So, the diameter is about 4 m.
The radius is $\frac{1}{2}$ the diameter. One-half of 4 m is 2 m.
So, the radius of the pool is about 2 m.
Since the calculated answers are close to the estimates, the answers are reasonable.

Practice

1. Calculate the circumference of each circle.
Give the answers to two decimal places.
Estimate to check the answers are reasonable.

a)


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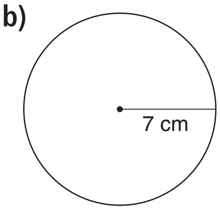

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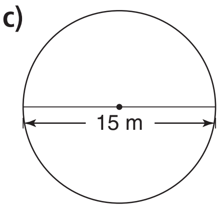
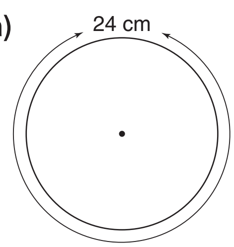


2. Calculate the diameter and radius of each circle.
Give the answers to two decimal places.
Estimate to check the answers are reasonable.

a)


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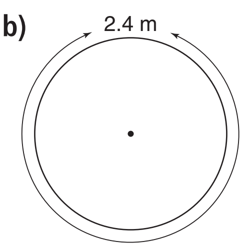

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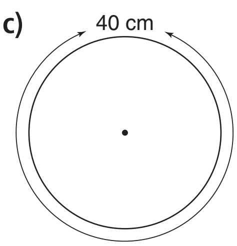

3. When you estimate to check the circumference, you use 3 instead of π.
Is the estimated circumference greater than or less than the actual circumference?
Why do you think so?

4. A circular garden has diameter 2.4 m.
a) The garden is to be enclosed with plastic edging.
How much edging is needed?
b) The edging costs $4.53/m.
What is the cost to edge the garden?

5. a) Suppose you double the diameter of a circle.
What happens to the circumference?
b) Suppose you triple the diameter of a circle.
What happens to the circumference?
Show your work.

6. A carpenter is making a circular tabletop with circumference 4.5 m.
What is the radius of the tabletop in centimetres?

Recall: 1 m = 100 cm

7. Can you draw a circle with circumference 33 cm?
If you can, draw the circle and explain how you know its circumference is correct.
If you cannot, explain why it is not possible.

8. **Assessment Focus** A bicycle tire has a spot of wet paint on it.
The radius of the tire is 46 cm.
Every time the wheel turns, the paint marks the ground.
a) What pattern will the paint make on the ground as the bicycle moves?
b) How far will the bicycle have travelled between two consecutive paint marks on the ground?
c) Assume the paint continues to mark the ground.
How many times will the paint mark the ground when the bicycle travels 1 km?
Show your work.

9. **Take It Further** Suppose a metal ring could be placed around Earth at the equator.
a) The radius of Earth is 6378.1 km. How long is the metal ring?
b) Suppose the length of the metal ring is increased by 1 km.
Would you be able to crawl under the ring, walk under the ring, or drive a school bus under the ring?
Explain how you know.

Reflect

What is π?
How is it related to the circumference, diameter, and radius of a circle?