2.5 Order of Operations with Integers

Focus Apply the order of operations with integers.

How many different ways can you evaluate this expression?
$9 \times 6 + 36 \div 4 - 1$

The expression can also be written as: $9(6) + \frac{36}{4} - 1$

To ensure everyone gets the same value, use the order of operations.

Recall the order of operations with whole numbers.

- Do the operations in brackets first.
- Multiply and divide, in order, from left to right.
- Add and subtract, in order, from left to right.

The same order of operations applies to all integers.

Investigate

Work in groups of 4.
Choose 5 different integers between –10 and +10.
Use any operations or brackets.
Find the expression that has the greatest integer value.
Find the expression that has the least integer value.

Reflect & Share

Trade expressions with another group of classmates.
Find the values of your classmates' expressions.
Check that you and your classmates get the same answers.

Connect

Since we use curved brackets to show an integer; for example, (–2), we use square brackets to group terms. For example, $[(+9) - (-2)] \times (-3)$

When an expression is written as a fraction, the fraction bar indicates division. The fraction bar also acts like a grouping symbol. That is, the operations in the numerator and the denominator must be done first before dividing the numerator by the denominator.

Example 1

Evaluate: $[(-6) + (-2)] \div (-4) + (-5)$

A Solution

$[(-6) + (-2)] \div (-4) + (-5)$ Do the operation in square brackets first.

$= (-8) \div (-4) + (-5)$ Divide.

$= (+2) + (-5)$ Add.

$= -3$

Example 2

Evaluate: $\dfrac{2 + 4 \times (-8)}{-6}$

A Solution

$\dfrac{2 + 4 \times (-8)}{-6}$ Evaluate the numerator.
Multiply.

$= \dfrac{2 + (-32)}{-6}$ Add.

$= \dfrac{-30}{-6}$ Divide.

$= 5$

If an integer does not have a sign, it is assumed to be positive; for example, 2 = +2. Then we do not need to put the number in brackets.

Example 3

Evaluate: $\dfrac{[18 - (-6)] \times (-2)}{3(-4)}$

A Solution

$\dfrac{[18 - (-6)] \times (-2)}{3(-4)}$ Evaluate the numerator and denominator separately.
Do the square brackets first.

$= \dfrac{24 \times (-2)}{3(-4)}$ Multiply.

$= \dfrac{-48}{-12}$ Divide.

$= 4$

Discuss the ideas

1. Why are the square brackets unnecessary in this expression?
$(-3) + [12 \div (-4)]$
2. In *Example 3*, why were the numerator and denominator evaluated separately?

Practice

Check

3. State which operation you do first.

a) $7 + (-1) \times (-3)$

b) $(-18) \div (-6) - (-4)$

c) $6 + (-4) - (-2)$

d) $(-2)[7 + (-5)]$

e) $(-3) \times (-4) \div (-1)$

f) $8 - 3 + (-4) \div (-1)$

4. Evaluate each expression in question 3. Show all steps.

5. Elijah evaluated this expression as shown.

$$\begin{aligned} 3 - (-5) + 8(-4) &= 3 - (-5) + (-32) \\ &= 3 - (-37) \\ &= 40 \end{aligned}$$

Is Elijah's solution correct? If your answer is yes, explain the steps Elijah took. If your answer is no, what error did Elijah make? What is the correct answer? Show your work.

6. **a)** Evaluate.

i) $12 \div (2 \times 3) - 2$

ii) $12 \div 2 \times (3 - 2)$

b) Why are the answers different? Explain.

Apply

7. Evaluate. State which operation you do first.

a) $7(4) - 5$ **b)** $6[2 + (-5)]$

c) $(-3) + 4(7)$ **d)** $(-6) + 4(-2)$

e) $15 \div [10 \div (-2)]$ **f)** $18 \div 2(-6)$

8. Evaluate. Show all steps.

a) $6(5 - 7) - 3$

b) $4 - [5 + (-11)]$

c) $[4 - (-8)] \div 6$

d) $8 - 66 \div (-11)$

e) $(-24) \div 12 + (-3)(-4)$

f) $6(-3) + (-8)(-4)$

9. Evaluate. Show all steps.

a) $\dfrac{(-7) \times 4 + 8}{4}$

b) $\dfrac{4 + (-36) \div 4}{-5}$

c) $\dfrac{-32}{(-6)(-2) - (-4)}$

d) $\dfrac{9}{(-3) + (-18) \div 3}$

10. Evaluate. Show all steps.

a) $\dfrac{4(-3) + 7(-4)}{5(-1)}$

b) $\dfrac{[19 - (-5)] \div (-3)}{2(-2)}$

c) $\dfrac{32 \div 4 - (-28) \div 7}{12 \div (-4)}$

d) $\dfrac{12 - 4(-6)}{[3 - (-3)] \times (-3)}$

11. **Assessment Focus** Robert, Brenna, and Christian got different answers for this problem: $(-40) - 2[(-8) \div 2]$
Robert's answer was –32, Christian's answer was –48, and Brenna's answer was 168.

a) Which student had the correct answer?

b) Show and explain how the other two students got their answers. What errors did they make?

12. Evaluate each expression. Then insert one pair of square brackets in each expression so it evaluates to –5.

a) $(-20) \div 2 - (-2)$

b) $(-21) + 6 \div 3$

c) $10 + 3 \times 2 - 7$

13. Keisha had $405 in her bank account. In one month, she made 4 withdrawals of $45 each. What is the balance in her account? Write an integer expression to represent this problem. Solve the problem. How did you decide which operations to use?

14. Use three –4s and any operations or brackets. Write an expression with a value of:

a) –12 **b)** –4 **c)** 0

d) –3 **e)** 5 **f)** 2

15. **Take It Further** The daily highest temperatures for one week in February were: –2°C, +5°C, –8°C, –4°C, –11°C, –10°C, –5°C
Find the mean highest temperature. How did you decide which operations to use?

16. **Take It Further** Write an expression for each statement. Evaluate each expression.

a) Divide the sum of –24 and 4 by –5.

b) Multiply the sum of –4 and 10 by –2.

c) Subtract 4 from –10, then divide by –2.

17. **Take It Further** Copy each equation. Replace each $\square$ with the correct sign $(+, -, \times, \div)$ to make each equation true.

a) $(-10) \square (-2) \square 1 = 21$

b) $(-5) \square (-2) \square 4 = 1$

c) $6 \square (-7) \square 2 = -44$

d) $(-2)(-2) \square 8 = -4$

Reflect

Suppose you evaluate an expression that has different operations.
How do you know where to begin? How do you know what to do next?
Make up an integer expression that has three operations.
Explain how you evaluate it.