

5.2

Calculating Percents

Focus Calculate percents from 0% to greater than 100%.

Have you ever used a photocopier to reduce or enlarge a picture?

To choose the size of the image picture, you select a percent.

Which percents might you choose if you want to reduce the picture?

Which percents might you choose if you want to enlarge the picture?

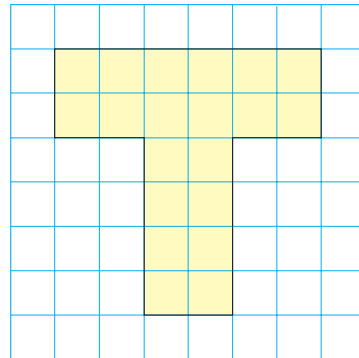


Investigate

Work with a partner.

Copy this shape.

- Redraw the shape so that each line segment is 150% of the length shown.
- Draw your own shape. Choose a different percent between 100% and 200%. Repeat the activity above.



Reflect & Share

Compare your drawings with those of another pair of classmates. What strategies did you use to create your enlargements? What do you notice about the lengths of corresponding line segments on the original shape and the enlarged shape?

Connect

Recall that when the whole is 1.0, you know that:

$$100\% = 1.0$$

$$10\% = 0.10$$

$$1\% = 0.01$$

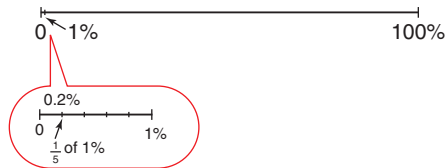
We can extend the pattern to write percents less than 1% as decimals:

$$0.1\% = 0.001$$

$$0.5\% = 0.005$$

We can use number lines to show percents between 0% and 1%.

For example, this number line shows 0.2%.



We can also extend the pattern to write percents greater than 100% as decimals.

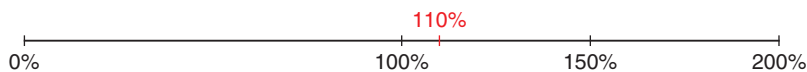
$$101\% = 1.01$$

$$110\% = 1.10, \text{ or } 1.1$$

$$150\% = 1.50, \text{ or } 1.5$$

$$200\% = 2.00, \text{ or } 2.0$$

We can use a number line to show percents greater than 100%.



Percents greater than 100% are used by store owners to calculate the prices of items they sell.

A store has to make a profit; that is, to sell goods for more than the goods cost to buy.

A store manager buys merchandise from a supplier.

The price the manager pays is called the *cost price*.

The manager *marks up* the cost price to arrive at the *selling price* for the customer.

The markup is the *profit*.

Cost price + Profit = Selling price

Example 1

- a) Write 210% as a decimal.
- b) Shade hundred charts to show 210%.

▶ A Solution

a) $210\% = \frac{210}{100}$

$$= 2.10, \text{ or } 2.1$$

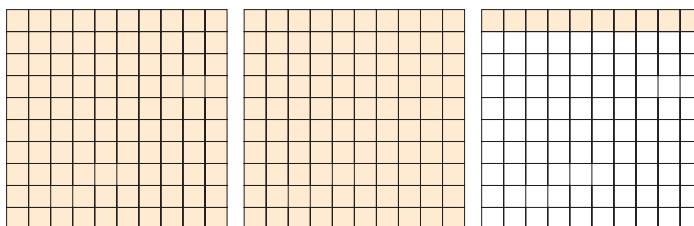
b) $210\% = 100\% + 100\% + 10\%$

Use a hundred chart to represent 100%.

To show 200%, shade all the squares in 2 hundred charts.

Each small square represents 1%.

So, to show 10%, shade 10 squares of a third hundred chart.



Example 2

The cost price of a winter coat is \$80.

The selling price of the coat is 230% of the cost price.

What is the selling price of the coat?

Illustrate the answer with a number line.

▶ A Solution

To find the selling price of the coat, find 230% of \$80.

First, write 230% as a decimal.

$$230\% = \frac{230}{100}$$

$$= 2.30, \text{ or } 2.3$$

$$\text{Then, } 230\% \text{ of } \$80 = 2.3 \times \$80$$

$$= \$184$$

The selling price of the coat is \$184.

We can show this answer on a number line.



Example 3

In 2004, the population of First Nations people living on reserves in Alberta was 58 782.

About 0.28% of these people belonged to the Mikisew Cree band.

- About how many people belonged to the Mikisew Cree band?
- Estimate to check the answer is reasonable.
- Illustrate the answer with a diagram.

A Solution

- a) Find 0.28% of 58 782.

First write 0.28% as a decimal.

$$\begin{aligned} 0.28\% &= \frac{0.28}{100} && \text{Multiply the numerator and the denominator by 100.} \\ &= \frac{28}{10\,000} \\ &= 0.0028 \end{aligned}$$

$$\begin{aligned} \text{Then, } 0.28\% \text{ of } 58\,782 &= 0.0028 \times 58\,782 && \text{Use a calculator.} \\ &= 164.5896 \end{aligned}$$

About 165 people belonged to the Mikisew Cree band.

- b) 0.28% is approximately 0.25%.

$$0.25\% \text{ is } \frac{1}{4}\%.$$

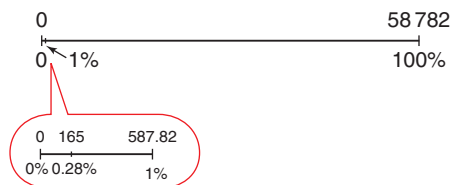
$$1\% \text{ of } 58\,782 \text{ is: } 0.01 \times 58\,782 = 587.82$$

587.82 is about 600.

$$600 \div 4 = 150$$

This estimate is close to the calculated answer, 165.

- c) To illustrate 0.28%, first show 1% on a number line.
Then, 0.28% is about $\frac{1}{4}$ of 1%.



Discuss the ideas

- As a decimal, $100\% = 1$.
What decimals correspond to percents greater than 100%?
What decimals correspond to percents less than 1%?
- In *Example 2*, how could you use the number line to find the profit?
- In *Example 2*, how could you estimate to check the answer?

Practice

Check

- 4.** A hundred chart represents 100%. Shade hundred charts to show each percent.
a) 150% b) 212% c) 300% d) 198%
- 5.** Write each percent as a decimal. Draw a diagram or number line to illustrate each percent.
a) 120% b) 250% c) 475%
d) 0.3% e) 0.53% f) 0.75%
- 6.** Write each decimal as a fraction and as a percent.
a) 1.7 b) 3.3 c) 0.003 d) 0.0056
- 7.** The cost price of a baseball cap is \$9. The selling price of the cap is 280% of the cost price. What is the selling price of the baseball cap? Illustrate the answer with a number line.

Apply

- 8.** What does it mean when someone states, “She gave it 110%”? How can this comment be explained using math? Is it possible to give 110%? Explain.
- 9.** a) Describe two situations when a percent may be greater than 100%.
b) Describe two situations when a percent may be between 0% and 1%.

- 10. a)** Write each fraction as a percent.

i) $\frac{1}{3}$ ii) $\frac{2}{3}$ iii) $\frac{3}{3}$

iv) $\frac{4}{3}$ v) $\frac{5}{3}$ vi) $\frac{6}{3}$

- b)** What patterns do you see in your answers in part a)?

- c)** Use these patterns to write each fraction as a percent.

i) $\frac{7}{3}$ ii) $\frac{8}{3}$ iii) $\frac{9}{3}$

iv) $\frac{10}{3}$ v) $\frac{11}{3}$ vi) $\frac{12}{3}$

- 11. a)** Find each percent of the number.

Draw a diagram to illustrate each answer.

i) 200% of 360 ii) 20% of 360

iii) 2% of 360 iv) 0.2% of 360

- b)** What patterns do you see in your answers in part a)?

- c)** Use the patterns in part a) to find each percent. Explain your work.

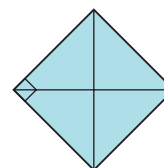
i) 2000% of 360 ii) 0.02% of 360

- 12.** A marathon had 618 runners registered. Of these runners, about 0.8% completed the race in under 2 h 15 min.

- a)** How many runners completed the race in this time?

- b)** Estimate to check your answer.

- 13. a)** This shape represents 100%. Draw a shape that represents 375%.



- b)** Repeat part a) using a shape of your own choice.

- 14.** The population of a small town in Alberta was 2600. The population increased by 5% one year and by 15% the next year. What was the town's population after the 2 years?
- To solve this problem, Juan calculated the population after a 5% increase. He then used his number to find the population after a 15% increase. What was Juan's answer?
 - To solve this problem, Jeremy calculated the population after a 20% increase. What was Jeremy's answer?
 - Compare your answers to parts a and b. Are Juan and Jeremy's answers the same? If your answer is yes, explain why both strategies work. If your answer is no, who is correct? Justify your choice.
- 15.** At the local theatre, 120 people attended the production of *Romeo and Juliet* on Friday. The attendance on Saturday was 140% of the attendance on Friday.
- How many people went to the theatre on Saturday?
 - Estimate to check your answer is reasonable.
- 16. Assessment Focus** During the 1888 Gold Rush, a British Columbia town had a population of about 2000. By 1910, the town had become a ghost town. The population was 0.75% of its population in 1888.
- Estimate the population in 1910. Justify your estimate.
 - Calculate the population in 1910.
 - Find the decrease in population from 1888 to 1910. Show your work.
- 17. Take It Further** Twenty boys signed up for the school play. The number of girls who signed up was 195% of the number of boys. At the auditions, only 26 girls attended. What percent of the girls who signed up for the play attended the auditions?
- 18. Take It Further** At an auction, a painting sold for \$148 500. This was 135% of what it sold for 3 years ago. What was the selling price of the painting 3 years ago? Justify your answer.
- 19. Take It Further** The perimeter of a rectangular window is 280% of its length. The length of the window is 145 cm. What is the width of the window? Show your work.

Reflect

How do you find a percent of a number in each case?

- The percent is less than 1%.
- The percent is greater than 100%.

Use an example to explain each case. Include diagrams.