5.1 Relating Fractions, Decimals, and Percents

The students in a First Nations school were asked which of 5 events at the Northern Manitoba Trappers' Festival they would most like to attend. The circle graph shows the results.
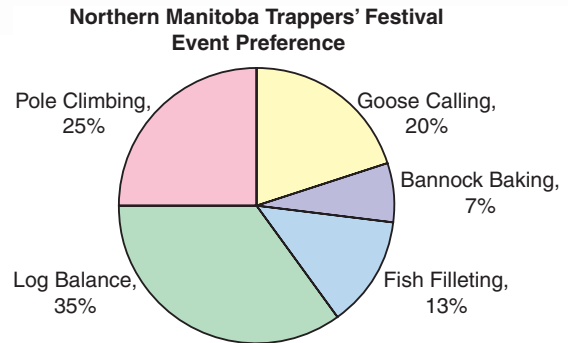
Which event was the favourite? How do you know? How else can you write that percent?

Northern Manitoba Trappers' Festival Event Preference

Pole Climbing, 25%
Goose Calling, 20%
Bannock Baking, 7%
Fish Filleting, 13%
Log Balance, 35%

Investigate

Work with a partner.
The Grades 7 and 8 students in 2 schools in Winnipeg were asked which of these cultural or historical attractions they would most like to visit.

Attraction	Number of Students	
	School A	School B
Winnipeg Aboriginal Centre	48	18
Chinese Cultural Centre	30	24
Upper Fort Gary Gate	39	3
Gabrielle Roy House	33	15

➤ How many students were surveyed in each school?
➤ Which school had the greater percent of students choosing the Winnipeg Aboriginal Centre? The Chinese Cultural Centre? Upper Fort Gary Gate? Gabrielle Roy House?

Reflect & Share

What strategies did you use to find the percents?
Compare your answers with those of another pair of classmates.
If the answers are different, how do you know which answers are correct?
At each school, what percent of students did not choose
Upper Fort Gary Gate? How did you find out?

Connect

To write a fraction as a percent, we first write the fraction
with a denominator that is a power of 10;
such as 10, 100, 1000, or 10 000.
Some fractions cannot be written this way.
Then, we can use a calculator to divide.

> A *power* of 10 is a product of any number of 10s.

➤ We can use a *hundred chart* to represent
one whole, or 100%.
Each small square represents 1%.

We can describe the shaded part
of the hundred chart in 3 ways:
as a percent, a decimal, and as a fraction.

There are $34\frac{1}{2}$ blue squares in 100 squares.

So, 34.5% of the squares are blue.

As a decimal: $\frac{34.5}{100} = 0.345$

> $\frac{1}{2} = 0.5$

As a fraction: Since the decimal has 3 digits after the decimal point, write a
fraction with denominator 1000.

$0.345 = \frac{345}{1000}$

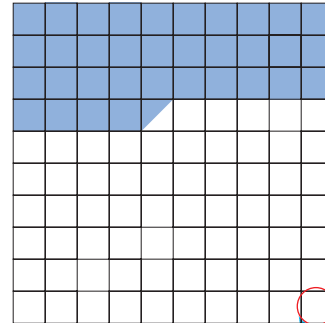$= \frac{345 \div 5}{1000 \div 5}$

$= \frac{69}{200}$

> Reduce to simplest form.
> 5 is a factor of both the
> numerator and the denominator.

➤ We can use a *hundredths chart* to represent 1%.
Each small square represents $\frac{1}{100}$ of 1%,

which we write as $\frac{1}{100}$%, or 0.01%.

To represent $\frac{1}{5}$ of 1%, or $\frac{1}{5}$% on the hundredths chart,

shade $\frac{1}{5}$ of the chart, which is 20 squares.
Since 1 small square is 0.01%, then 20 small squares
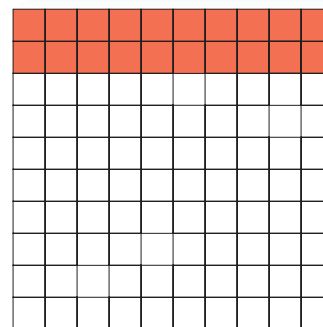are 0.20%, or 0.2%.

We can write this percent as a decimal.

$0.2\% = \frac{0.2}{100} = \frac{2}{1000} = 0.002$

> Be careful not to confuse
> a decimal percent, such as 0.2%,
> with the decimal 0.2,
> which is 20%.

5.1 Relating Fractions, Decimals, and Percents   **235**

Example 1

Write each percent as a fraction and as a decimal.

a) 7%   **b)** 7.75%   **c)** $7\frac{1}{4}\%$

▶ *A Solution*

a) $7\% = \frac{7}{100}$

$\quad\quad = 0.07$

b) $7.75\% = \frac{7.75}{100}$ Multiply the numerator and the denominator by 100.

$\quad\quad\quad = \frac{775}{10\,000}$

$\quad\quad\quad = 0.0775$

Write the fraction in simplest form.

$\frac{775}{10\,000} = \frac{775 \div 25}{10\,000 \div 25}$ 25 is a factor of both the
numerator and the denominator.
$\quad\quad\quad = \frac{31}{400}$ So, divide by 25.

c) $7\frac{1}{4}\% = \frac{7.25}{100}$ $\boxed{\frac{1}{4} = 0.25}$

$\quad\quad\quad = \frac{725}{10\,000}$

$\quad\quad\quad = 0.0725$

Write the fraction in simplest form.

$\frac{725}{10\,000} = \frac{725 \div 25}{10\,000 \div 25}$

$\quad\quad\quad = \frac{29}{400}$

We can show each decimal in *Example 1* in a place-value chart.

Ones	Tenths	Hundredths	Thousandths	Ten Thousandths
0	0	7	0	0
0	0	7	7	5
0	0	7	2	5

Example 2

Write each fraction as a decimal and as a percent.

a) $\frac{5}{8}$ b) $\frac{5}{6}$ c) $\frac{5}{1000}$

▶ A Solution

a) $\frac{5}{8}$ means $5 \div 8$.   Use a calculator.

$\frac{5}{8} = 0.625$

$0.625 = \frac{625}{1000}$ Divide the numerator and the denominator by 10 to get an equivalent fraction with denominator 100.

$ = \frac{625 \div 10}{1000 \div 10}$

$ = \frac{62.5}{100}$

$ = 62.5\%$

b) $\frac{5}{6} = 5 \div 6$ Use a calculator.

$\phantom{\frac{5}{6}} = 0.8\overline{3}$

This is a repeating decimal.

> **Recall that the bar over the 3 indicates that the digit repeats.**

To write an equivalent fraction with denominator 100, first write $0.8\overline{3}$ as $0.83\overline{3}$.

$0.83\overline{3} = \frac{83.\overline{3}}{100}$

$\phantom{0.83\overline{3}} = 83.\overline{3}\%$

c) $\frac{5}{1000} = 5 \div 1000$

$\phantom{\frac{5}{1000}} = 0.005$

Divide the numerator and the denominator by 10 to get an equivalent fraction with denominator 100.

$\frac{5}{1000} = \frac{5 \div 10}{1000 \div 10}$

$\phantom{\frac{5}{1000}} = \frac{0.5}{100}$

$\phantom{\frac{5}{1000}} = 0.5\%$

We can use decimals or percents to compare two test marks when the total marks are different.

Example 3

Buffy had $23\frac{1}{2}$ out of 30 on her first math test.

She had $31\frac{1}{2}$ out of 40 on her second math test.

On which test did Buffy do better?

▶ *A Solution*

Write each test mark as a percent.
First test:

$23\frac{1}{2}$ out of 30 $= \frac{23.5}{30}$ Divide. Use a calculator.

$= 0.78\bar{3}$

Since the decimal has 3 digits after the decimal point,
write a fraction with denominator 1000.

$0.78\bar{3} = \frac{783.\bar{3}}{1000}$

$= \frac{783.\bar{3} \div 10}{1000 \div 10}$ Divide the numerator and the denominator by 10 to get an equivalent fraction with denominator 100.

$= \frac{78.\bar{3}}{100}$

$= 78.\bar{3}$ %

Second test:

$31\frac{1}{2}$ out of 40 $= \frac{31.5}{40}$ Use a calculator.

$= 0.7875$

Since the decimal has 4 digits after the decimal point,
write a fraction with denominator 10 000.

$0.7875 = \frac{7875}{10\,000}$ Divide the numerator and the denominator by 100.

$= \frac{7875 \div 100}{10\,000 \div 100}$

$= \frac{78.75}{100}$

$= 78.75\%$

Since $78.75\% > 78.\bar{3}\,\%$, Buffy did better on the second test.

1. How can we use a grid with 100 squares to show 100% and also to show 1%?

2. How can we use a grid with 100 squares to show 0%? Justify your answer.

3. Explain why $\frac{1}{5}$ and $\frac{1}{5}\%$ represent different numbers.

4. In *Example 1*, we simplified $\frac{775}{10\,000}$ by dividing the numerator and denominator by 25.

   a) Why did we choose 25?

   b) Could we have simplified the fraction a different way? Explain.

5. In *Example 3*, how could we solve the problem without finding percents?

Practice

Check

6. Each hundred chart represents 100%. What fraction of each hundred chart is shaded? Write each fraction as a decimal and as a percent.
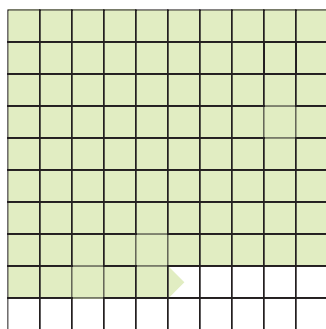
   a)

   b)

   c)

   d)
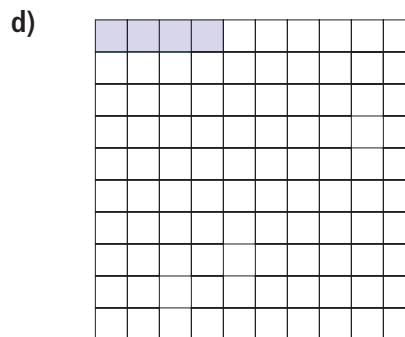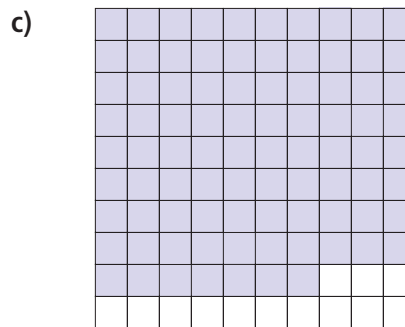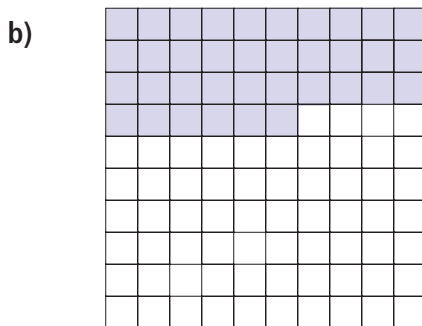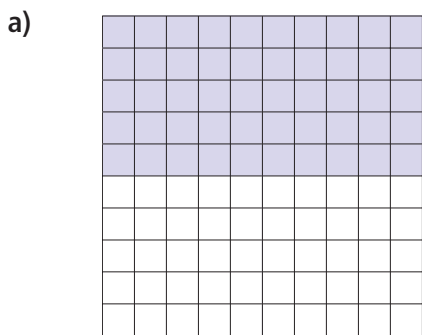
7. Write each percent as a fraction and as a decimal.

   a) 3%   b) 51%   c) 98%   d) 29%

8. Each hundred chart represents 100%. What fraction of each hundred chart is shaded? Write each fraction as a decimal and as a percent.
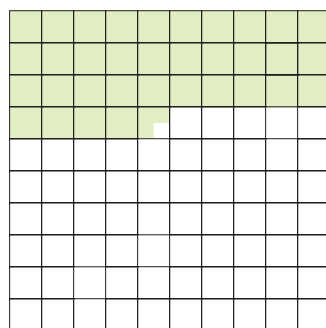
a)
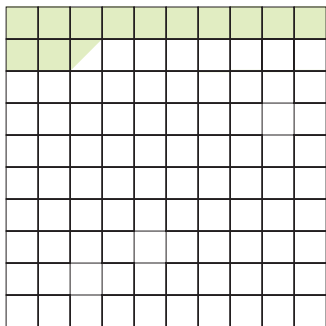
b)

c)

Apply

9. Write each percent as a fraction and as a decimal.
a) 73.5%   **b)** 21.25%
c) $8\frac{3}{4}\%$ **d)** $1\frac{1}{5}\%$

10. Use a hundredths chart to represent 1%. Shade the chart to represent each percent.
a) 0.75%   **b)** 0.4%
c) 0.07%   **d)** 0.95%

11. Use a hundredths chart to represent 1%. Shade the chart to represent each percent.
a) 0.655%   **b)** 0.0225%
c) $\frac{2}{3}\%$ **d)** $\frac{2}{5}\%$

12. Write each percent as a fraction and as a decimal.
a) 0.25%   **b)** 0.6%
c) 0.5%   **d)** 0.38%

13. Write each fraction as a decimal and as a percent.
a) $\frac{2}{300}$ **b)** $\frac{18}{400}$
c) $\frac{7}{500}$ **d)** $\frac{8}{250}$

14. Write each decimal as a fraction and as a percent.
a) 0.345   **b)** 0.0023
c) 0.1825   **d)** 0.007

15. A hundredths chart represents 1%. Forty-five of its squares are shaded. Arjang says the shaded squares represent $\frac{45}{100}$. Fiona says the shaded squares represent 0.0045. Who is correct? Write to explain where the other student went wrong.

16. Vince scored 82.5% on a math test. Junita had 15 out of 18 on the same test. Who did better? How do you know?

17. Suppose you were asked to tutor another student.
   a) i) How would you explain $\frac{5}{8}$ as a fraction?
     ii) What real-life example could you use to help?
   b) i) How would you explain $\frac{5}{8}$ as a quotient?
     ii) What real-life example could you use to help?

18. Assessment Focus You will need 1-cm grid paper and coloured pencils.
   a) Draw a 6-cm by 8-cm rectangle. Shade:
     • $33.\overline{3}$ % of the grid squares in the rectangle red
     • 0.25 of the grid squares green
     • $\frac{3}{8}$ of the grid squares blue
     Explain how you did this.
   b) What fraction of the rectangle is not shaded? Write this fraction as a decimal and as a percent.
   c) Do you think you could have completed part a with a 6-cm by 9-cm rectangle? With a square of side length 7 cm? Explain.

19. A student council representative is elected from each homeroom class in the school. Joanna received 23 of 30 votes in her class. Kyle received 22 of 28 votes in his class. Who received the greater percent of votes, Joanna or Kyle? How did you find out?

20. A student used this strategy to write $6\frac{1}{4}$% as a fraction.

$$6\frac{1}{4}\% = 6.25\%$$
$$= \frac{625}{100}$$
$$= \frac{625 \div 25}{100 \div 25}$$
$$= \frac{25}{4}$$

   a) Check the student's work. Is the strategy correct?
   b) If your answer is yes, write the fraction as a decimal. If your answer is no, describe the error then correct it.

21. Take It Further Replace each ☐ with <, >, or = to make each statement true.
   a) 3.21 ☐ 321%
   b) $1\frac{5}{8}$ ☐ 158%
   c) 0.76 ☐ 7.6%
   d) 0.9% ☐ 0.9
   e) $0.\overline{3}$ % ☐ $\frac{1}{3}$%
   f) 125% ☐ $1\frac{1}{4}$

Reflect

What did you know about fractions, decimals, and percents before you began this lesson? What do you know about fractions, decimals, and percents now?