

2.3

Using Models to Divide Integers

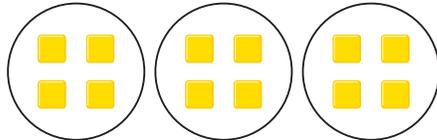
Focus Use a model to divide integers.

We can think of division as the opposite of multiplication.

$$12 \div 4 = ?$$

This can mean how many sets of 4 will give a product of 12:

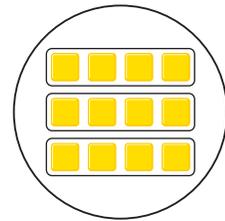
$$? \times 4 = 12$$



There are 3 sets of 4.
So, $3 \times 4 = 12$

We can use a “bank” model to multiply 2 integers.

- A circle represents the “bank.”
- We start with the bank having zero value.
- The first integer tells us to deposit (put in) or to withdraw (take out).
- The second integer tells us what to put in or take out.
- We can use this model to multiply 3×4 .



Make 3 deposits of 4 yellow tiles.
There are 12 yellow tiles.
So, $3 \times 4 = 12$

How can we use this model to find $12 \div 4$?

Investigate

Work with a partner.

You will need coloured tiles.

Choose 2 positive integers between 1 and 20 whose quotient is an integer.

For example: 12 and 3, since $12 \div 3 = 4$

Use these integers and their opposites.

Write all possible division expressions.

Use the tiles and a “bank” model to divide.

Sketch the tiles you used in each case.

Write a division equation each time.

A quotient is the number that results from the division of one number by another.



Reflect & Share

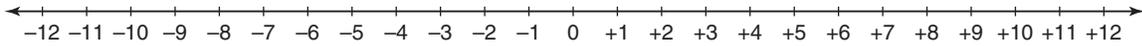
Share your results with the class.

What patterns do you notice?

How can you predict the quotient of two integers?

Connect

We can extend the use of a number line to model the division of two integers.



Visualize walking the line to divide integers.

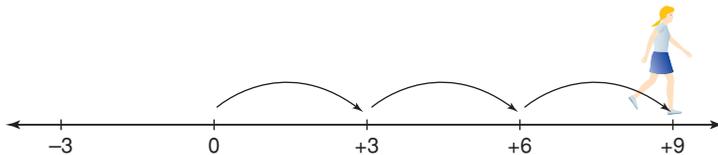
This time, the direction you end up facing determines the sign of the quotient.

► Divide: $(+9) \div (+3)$

We need to find how many steps of $+3$ make $+9$.

The step size, $+3$, is positive; so, walk forward.

Start at 0. Take steps forward to end up at $+9$.



We took 3 steps. We are facing the positive end of the line.

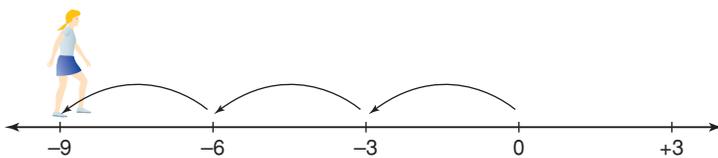
So, $(+9) \div (+3) = +3$

► Divide: $(-9) \div (-3)$

We need to find how many steps of -3 make -9 .

The step size, -3 , is negative; so, walk backward.

Start at 0. Take steps backward to end up at -9 .



We took 3 steps. We are facing the positive end of the line.

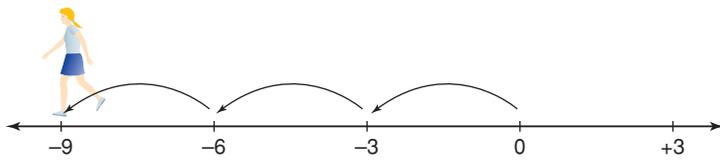
So, $(-9) \div (-3) = +3$

► Divide: $(-9) \div (+3)$

We need to find how many steps of $+3$ take us to -9 .

The step size, $+3$, is positive; so, walk forward.

Start at 0. To end up at -9 , we took 3 steps forward.



We are facing the negative end of the line.

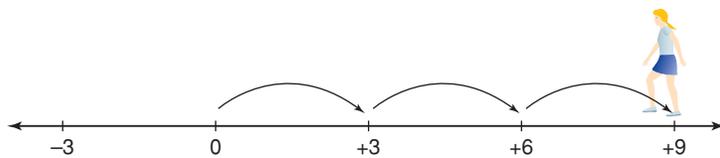
So, $(-9) \div (+3) = -3$

► Divide: $(+9) \div (-3)$

We need to find how many steps of -3 take us to $+9$.

The step size, -3 , is negative; so, walk backward.

Start at 0. To end up at $+9$, we took 3 steps backward.



We are facing the negative end of the line.

So, $(+9) \div (-3) = -3$

Example 1

The 1850-km Iditarod dogsled race lasts from 10 to 17 days.

One night, the temperature fell 2°C each hour for a total change of -12°C .

Use integers to find how many hours this change in temperature took.

A Solution

-2 represents a fall of 2°C .

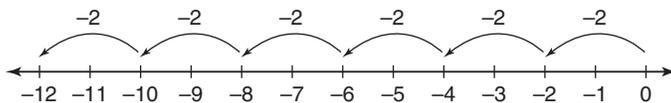
-12 represents a change of -12°C .

Using integers, we need to find how many -2 s take us to -12 ; that is, $(-12) \div (-2)$.

Start at 0.

Move 2 units left.

Continue to move 2 units left until you reach -12 .



Six moves of 2 units left were made.

So, $(-12) \div (-2) = +6$



Example 2

Use a model to find the quotient: $(-12) \div (+4)$

A Solution

Divide: $(-12) \div (+4)$

How many groups of $+4$ will make -12 ?

Use coloured tiles and the “bank” model.

Start with a value of 0 in the circle.

To get a product of -12 , 12 red tiles must be left in the circle.

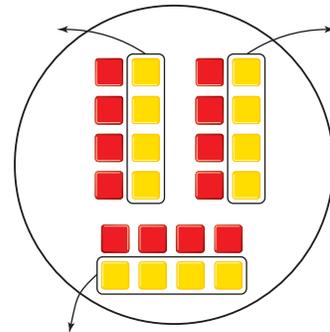
So, model 0 with 12 zero pairs.

$+4$ is modelled with 4 yellow tiles.

Take out sets of 4 yellow tiles.

3 sets were removed.

So, $(-12) \div (+4) = -3$



The negative sign in the quotient indicates removal.

Discuss the ideas

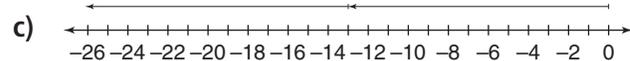
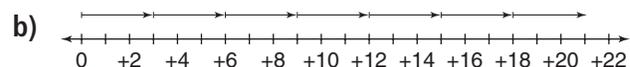
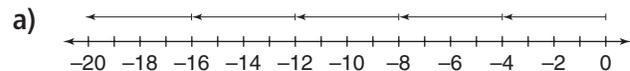
- Which model do you prefer to use to divide integers?
- How can you use the inverse operation to check your answers?

Practice

Check

- Write a related multiplication equation for each division equation.
 - $(+25) \div (+5) = +5$
 - $(+24) \div (-2) = -12$
 - $(-14) \div (-7) = +2$
 - $(-18) \div (+6) = -3$

- Which integer division does each number line represent? Find each quotient.



5. Enrico walked a number line to model a division. He started at 0. Enrico took steps forward of size 4. He ended up at -24 . Which division did Enrico model? How did you find out?

6. Use a number line. Find each quotient.

- a) $(+8) \div (+1)$ b) $(-6) \div (-2)$
 c) $(-16) \div (+8)$ d) $(-3) \div (-1)$
 e) $(+15) \div (-3)$ f) $(-20) \div (+2)$

7. a) How many sets?

- i) 12 yellow tiles grouped in sets of 6
 ii) 15 red tiles grouped in sets of 3
 b) How many in each set?
 i) 8 yellow tiles shared among 2 sets
 iii) 21 red tiles shared among 7 sets

Apply

8. Use coloured tiles to represent each division. Find each quotient. Sketch the tiles you used.

- a) $(+18) \div (+6)$ b) $(-18) \div (+9)$
 c) $(-16) \div (-4)$ d) $(+21) \div (-7)$
 e) $(+15) \div (-5)$ f) $(-16) \div (-8)$

9. Use coloured tiles, a number line, or another model to show your thinking clearly. Find each quotient.

- a) $(+8) \div (+4)$ b) $(-8) \div (-4)$
 c) $(+8) \div (-4)$ d) $(-8) \div (+4)$

Compare the quotients. What do you notice?

10. Use coloured tiles, a number line, or another model to show your thinking clearly. Find each quotient.

- a) $(+24) \div (+8)$ b) $(-20) \div (-5)$
 c) $(+28) \div (-7)$ d) $(-25) \div (+5)$
 e) $(-14) \div (+2)$ f) $(+18) \div (-9)$

11. The temperature rose 3°C each hour for a total change of $+12^{\circ}\text{C}$. Use integers to find the number of hours the change in temperature took.

12. The temperature fell 4°C each hour for a total change of -20°C . Use integers to find the number of hours the change in temperature took.

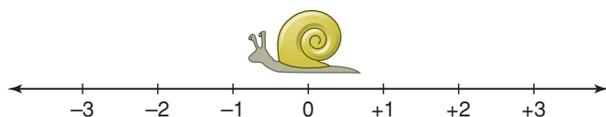
13. A submarine was at the surface of the ocean. It made 4 identical plunges in a row. Its final depth was 148 m below sea level. What was the depth of each plunge?



14. **Assessment Focus** Heather used the expression $(+45) \div (-5)$ to solve a word problem. What might the word problem have been? Show as many different ways as you can to solve the problem.

15. Maddie used the expression $(-12) \div (+6)$ to solve a word problem. What might the word problem have been? Solve the problem.

- 16.** A snail travels along a number line marked in centimetres. A distance of 1 cm to the right is represented by +1. A distance of 1 cm to the left is represented by -1. The snail moves 6 cm to the left each minute.



- a) The snail is at 0 now. After how many minutes will the snail be at -36 on the number line?
- b) When was the snail at +18 on the number line?

Draw a model to represent each answer. Write a division equation for each model.

- 17. Take It Further** Abraham used the Internet to find the low temperature in six Western Canadian cities on a particular day in January. He recorded the temperatures in a table.

City	Low Temperature (°C)
Whitehorse	-10
Iqaluit	-7
Vancouver	+9
Edmonton	-1
Winnipeg	-9
Saskatoon	-6

- a) Find the mean low temperature for these cities on that day.
- b) The low temperature in Regina for the same day was added to the table. The mean low temperature for the seven cities was -3°C . What was the temperature in Regina?

Communicate your thinking clearly.

- 18. Take It Further** Reena deposits \$4 into her savings account each week. Today, Reena's account has a balance of \$16.

- a) How many weeks from now will Reena's account have a balance of \$40?
- b) What was the balance in Reena's account 2 weeks ago?

Explain how you can use integers to model each situation.

Reflect

How is the division of integers similar to the division of whole numbers? How is it different? Use examples to explain.