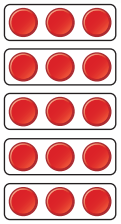
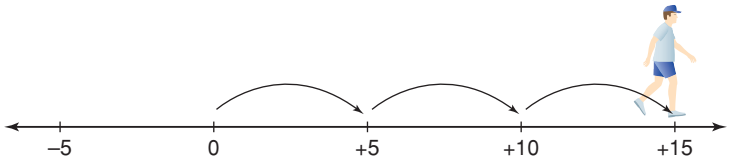
2.1 Using Models to Multiply Integers

Focus Use a model to multiply two integers.

We can think of multiplication as repeated addition.
5×3 is the same as adding five 3s: $3 + 3 + 3 + 3 + 3$
As a sum: $3 + 3 + 3 + 3 + 3 = 15$
As a product: $5 \times 3 = 15$



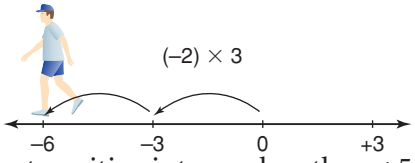
How can we think of 3×5?
One way is to use a number line.
Take 3 steps each of size 5.
So, $3 \times 5 = 15$

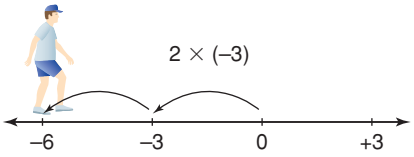


Investigate

Work with a partner.
You will need masking tape and a metre stick.
Make a large number line across the floor.
Divide the line into intervals of 15 cm.
Label the line from –15 to +15.

➤ Walk the line to multiply integers.
– Start at 0.
– For negative numbers of steps, face the negative end of the line before walking.
– For negative step sizes, walk backward.
For example:





➤ Choose 2 different positive integers less than +5.

➤ Find all possible products of the integers and their opposites.
Take turns. One partner walks the line to find each product.
The other partner records on a number line and writes the multiplication equation each time.

Reflect & Share

Share your results with the class. What patterns do you notice?
How can you predict the product of two integers?

Connect

Recall that we can use coloured tiles to model integers.
One yellow tile models +1, and one red tile models –1.

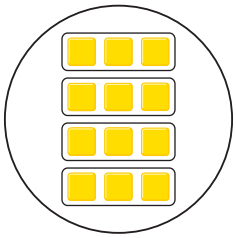

They combine to form a zero pair: (+1) + (–1) = 0

We can extend our use of coloured tiles to model the multiplication of two integers.
Let a circle represent the "bank." Start with the bank having zero value.
The first integer tells us to deposit (put in) or to withdraw (take out).
When the first integer is positive, put tiles in.
When the first integer is negative, take tiles out.
The second integer tells us what to put in or take out.

➤ Multiply: (+4) × (+3)
+4 is a positive integer.
+3 is modelled with 3 yellow tiles.
So, *put* 4 sets of 3 yellow tiles *into* the circle.

There are 12 yellow tiles in the circle.
They represent +12.
So, (+4) × (+3) = +12

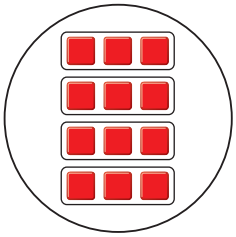
(+4) × (+3) = (+3) + (+3) + (+3) + (+3)
Make 4 deposits of +3.



➤ Multiply: (+4) × (–3)
+4 is a positive integer.
–3 is modelled with 3 red tiles.
So, *put* 4 sets of 3 red tiles *into* the circle.

There are 12 red tiles in the circle.
They represent –12.
So, (+4) × (–3) = –12

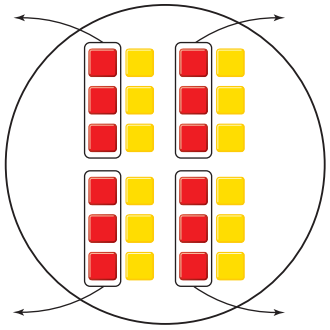
(+4) × (–3) = (–3) + (–3) + (–3) + (–3)
Make 4 deposits of –3.



➤ Multiply: (–4) × (–3)
–4 is a negative integer.
–3 is modelled with 3 red tiles.
So, *take* 4 sets of 3 red tiles *out* of the circle.
There are no red tiles to take out.

Make 4 withdrawals of –3.

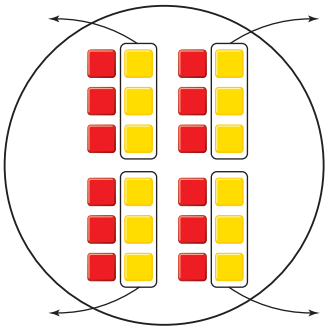
So, add zero pairs until there are enough red tiles to remove.
Add 12 zero pairs.
Take out 4 sets of 3 red tiles.



There are 12 yellow tiles left in the circle.
They represent +12.
So, $(-4) \times (-3) = +12$

➤ Multiply: $(-4) \times (+3)$
–4 is a negative integer.
+3 is modelled with 3 yellow tiles.
So, take 4 sets of 3 yellow tiles out of the circle.
There are no yellow tiles to take out.
So, add zero pairs until there are enough yellow tiles to remove.
Add 12 zero pairs.
Take out 4 sets of 3 yellow tiles.

Make 4 withdrawals of +3.

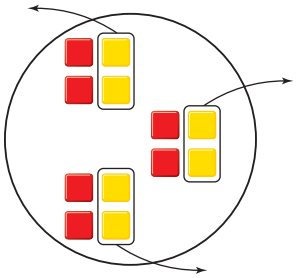


There are 12 red tiles left in the circle.
They represent –12.
So, $(-4) \times (+3) = -12$

Example 1

Use tiles to find: $(-3) \times (+2)$

A Solution



–3 is a negative integer.
+2 is modelled with 2 yellow tiles.
So, take 3 sets of 2 yellow tiles out of the circle.
Since there are no yellow tiles to take out, add 6 zero pairs.
Take out 3 sets of 2 yellow tiles.
6 red tiles remain.
They represent –6.
So, $(-3) \times (+2) = -6$

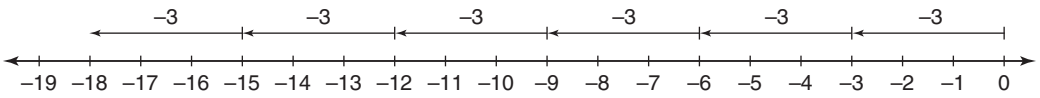
Example 2

The temperature fell 3°C each hour for 6 h.
Use an integer number line to find the total change in temperature.

A Solution

–3 represents a fall of 3°C.
+6 represents 6 h.
Using integers, we need to find: $(+6) \times (-3)$
$(+6) \times (-3) = (-3) + (-3) + (-3) + (-3) + (-3) + (-3)$
Use a number line. Start at 0.
Move 3 units left, 6 times.

To add a negative integer, move left on a number line.



So, $(+6) \times (-3) = -18$
The total change in temperature is –18°C.

Discuss the ideas

1. Which model do you find easiest to use to multiply integers?
2. What other models can you think of?
3. What do you notice about the effect of the order of the integers on the product?
4. Use a model to explain multiplication of an integer by 0.

Math Link

Sports

In hockey, each player has a plus/minus statistic. A player's plus/minus statistic increases by 1 when his team scores a goal while he is on the ice. A player's plus/minus statistic decreases by 1 when his team is scored against while he is on the ice. For example, a player begins a game with a plus/minus statistic of –7. During the game, his team scores 3 goals while he is on the ice and the opposing team scores 1 goal. What is the player's new plus/minus statistic?

Practice

Check

5. Write a multiplication expression for each repeated addition.

a) $(-1) + (-1) + (-1)$

b) $(-2) + (-2) + (-2) + (-2) + (-2)$

c) $(+11) + (+11) + (+11) + (+11)$

6. Write each multiplication expression as a repeated addition. Use coloured tiles to find each sum.

a) $(+7) \times (-4)$

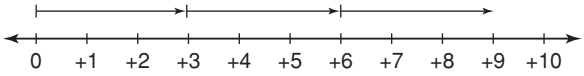
b) $(+6) \times (+3)$

c) $(+4) \times (+6)$

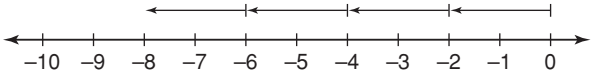
d) $(+5) \times (-6)$

7. Which integer multiplication does each number line represent? Find each product.

a)



b)



8. Use a number line. Find each product.

a) $(+6) \times (-1)$

b) $(+3) \times (+9)$

c) $(+2) \times (+6)$

d) $(+4) \times (-5)$

9. Which product does each model represent? Write a multiplication equation for each model.

a) Deposit 5 sets of 2 red tiles.

b) Deposit 5 sets of 2 yellow tiles.

c) Withdraw 7 sets of 3 red tiles.

d) Withdraw 9 sets of 4 yellow tiles.

e) Deposit 11 sets of 3 yellow tiles.

f) Withdraw 10 sets of 5 red tiles.

Apply

10. Use a circle and coloured tiles. Find each product. Sketch the tiles you used.

a) $(+1) \times (+5)$ **b)** $(+8) \times (+3)$

c) $(+7) \times (-2)$ **d)** $(+8) \times (-3)$

e) $(-5) \times (+6)$ **f)** $(-4) \times (-8)$

11. Use coloured tiles or a number line. Find each product.

a) $(+4) \times (+2)$ **b)** $(-4) \times (-2)$

c) $(+2) \times (+8)$ **d)** $(+5) \times (-6)$

e) $(-4) \times (+6)$ **f)** $(-7) \times (-3)$

12. The temperature rose 2°C each hour for 9 h. Use integers to find the total change in temperature.

13. Donovan was draining an above-ground swimming pool. The water level dropped 3 cm each hour for 11 h. Use integers to find the change in the water level after 11 h.

14. Lissa used the expression $(+8) \times (-6)$ to solve a word problem.

a) What might the word problem have been? Solve the problem.

b) Compare your word problem with those of your classmates. Which is your favourite problem?

15. **Assessment Focus** How many different ways can you model the product $(-7) \times (-8)$?
Show each strategy. Which strategy do you prefer? Explain your choice.

16. Rema was playing a board game. She moved back 4 spaces on each of 4 consecutive turns. Use integers to find her total change in position.

17. Ellen's dad spends $5 a week on newspapers.

a) How much less newspaper money will he have 8 weeks from now?

b) How much more newspaper money did he have 2 weeks ago?

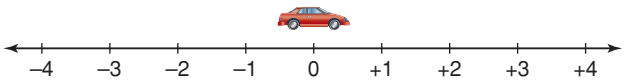
Draw a model to represent each answer. Write the equation that each model represents.

18. A toy car travels along a number line marked in centimetres. A distance of 1 cm to the right is represented by +1. A distance of 1 cm to the left is represented by –1. The car moves 4 cm to the left each second.

a) The car is at 0 now. Where will the car be 10 s from now?

b) Where was the car 3 s ago?

c) How can you use an operation with integers to answer parts a and b?



19. Hugh used the expression $(-7) \times (+6)$ to solve a word problem. What might the word problem have been? Solve the problem.

20. **Take It Further** Use coloured tiles or a number line. Find each product.

a) $(+3) \times (-2) \times (+4)$

b) $(-5) \times (-1) \times (+3)$

c) $(-5) \times (-2) \times (-3)$

d) $(+2) \times (-3) \times (-6)$

Reflect

How did your knowledge of adding integers help you with this lesson?
How did your knowledge of opposites change with this lesson?