

7.6

Tree Diagrams

Focus Investigate outcomes of probability experiments.

Recall that an outcome is the possible result of an experiment or action. When you roll a die, the outcomes are equally likely. When you toss a coin, the outcomes are equally likely. Some experiments have two or more actions.

Explore



You will need a die labelled 1 to 6 and a coin.

- List the possible outcomes of rolling the die and tossing the coin.
How many possible outcomes are there?
How many outcomes include rolling a 4? Tossing a head?
- What is the theoretical probability of the event “a head on the coin and a 2 on the die?”
- Conduct the experiment.
One of you tosses the coin and one rolls the die.
Record the results.
Calculate the experimental probability of the event “a head on the coin and a 2 on the die” after each number of trials.
 - 10 trials
 - 20 trials
 - 50 trials
 - 100 trials
- How do the experimental and theoretical probabilities compare?



Reflect & Share

Compare the strategy you used to find the outcomes with that of another pair of classmates. Was one strategy more efficient than another? Explain. Compare your probabilities. Combine your results to get 200 trials. What is the experimental probability of the event “a head on the coin and a 2 on the die?” How do the experimental and theoretical probabilities compare?

Connect

Two events are **independent events** if the result of the one event does not depend on the result of the other event.

Tossing two coins is an example of two independent events.

The outcome of the first toss does not affect the outcome of the second toss.

The outcome of the second toss does not depend on the outcome of the first toss.

We can use a **tree diagram** to show the possible outcomes for an experiment that has two independent events.

When 2 coins are tossed, the outcomes for each coin are heads (H) or tails (T).

List the outcomes of the first coin toss.

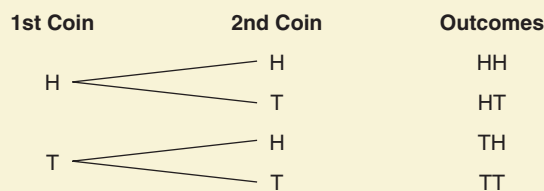
This is the first branch of the tree diagram.

For each outcome, list the outcomes of the second coin toss.

This is the second branch of the tree diagram.

Then list the outcomes for the coins tossed together.

You could also use a table to list the outcomes.



		First Coin	
		H	T
Second Coin	H	HH	HT
	T	TH	TT

There are 4 possible outcomes: HH, HT, TH, TT

Example

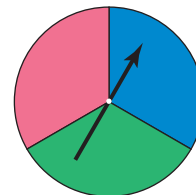
On this spinner, the pointer is spun once.

The colour is recorded.

The pointer is spun a second time.

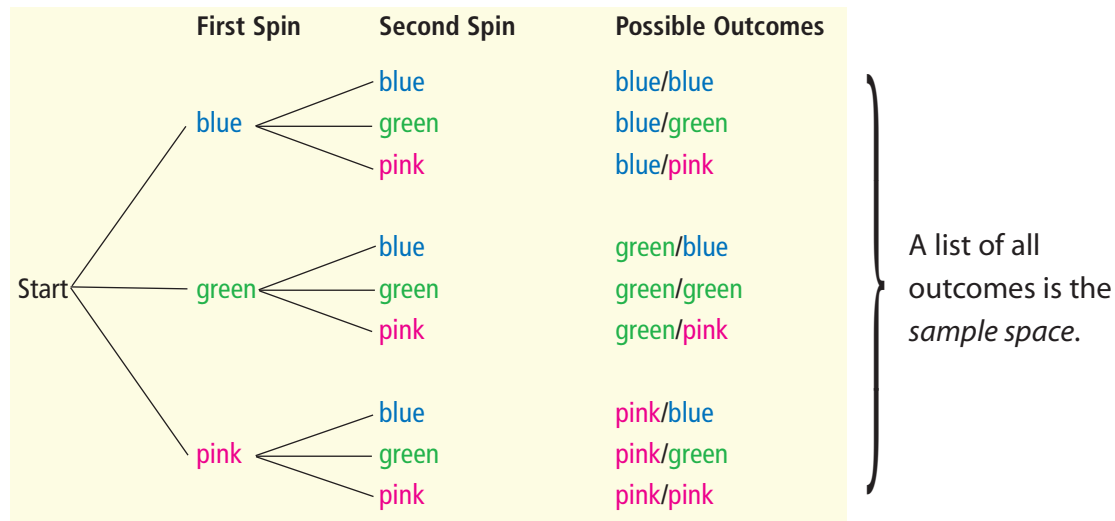
The colour is recorded.

- Draw a tree diagram to list the possible outcomes.
- Find the probability of getting the same colours.
- Find the probability of getting different colours.
- Carina and Paolo carry out the experiment 100 times. There were 41 same colours and 59 different colours. How do the experimental probabilities compare to the theoretical probabilities? Explain.



A Solution

- a) The first branch of the tree diagram lists the equally likely outcomes of the first spin: blue, green, pink
The second branch lists the equally likely outcomes of the second spin: blue, green, pink
For each outcome from the first spin, there are 3 possible outcomes for the second spin.
Follow the paths from left to right. List all the possible outcomes.



- b) From the tree diagram, there are 9 possible outcomes.
3 outcomes have the same colours: blue/blue, green/green, pink/pink
The probability of the same colour is:
 $\frac{3}{9} = \frac{1}{3} \doteq 0.33$, or about 33%
- c) 6 outcomes have different colours: blue/green, blue/pink, green/blue, green/pink, pink/blue, pink/green
The probability of different colours is:
 $\frac{6}{9} = \frac{2}{3} \doteq 0.67$, or about 67%
- d) The experimental probability of the same colour is:
 $\frac{41}{100} = 0.41$, or 41%
The experimental probability of different colours is:
 $\frac{59}{100} = 0.59$, or 59%
These probabilities are different from the theoretical probabilities.
The greater the number of times the experiment is carried out, the closer the theoretical and experimental probabilities may be.

Practice

1. List the sample space for each pair of independent events.

Why are the events independent?

- a) Rolling a die labelled 3 to 8 and tossing a coin



- b) Rolling a tetrahedron labelled 1 to 4 and spinning a pointer on this spinner

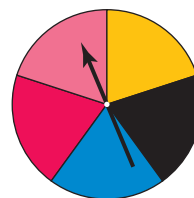
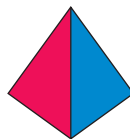


- c) Rolling a pair of dice labelled 1 to 6



2. Use the outcomes from question 1a. Aseer wins if an odd number or a head shows. Roberto wins if a number less than 5 shows. Who is more likely to win? Explain.
3. Use the outcomes from question 1b. Name an outcome that occurs about one-half of the time.
4. Use the outcomes from question 1c. How often are both numbers rolled greater than 4?
5. Hyo Jin is buying a new mountain bike. She can choose from 5 paint colours—black, blue, red, silver, or gold—and 2 seat colours—grey or black.
- a) Use a table to display all the possible combinations of paint and seat colours.
- b) Suppose Hyo Jin were to choose colours by pointing at lists without looking. What is the probability she would end up with a silver or black bike with a grey seat?

- 6. Assessment Focus** Tara designs the game *Mean Green Machine*. A regular tetrahedron has its 4 faces coloured red, pink, blue, and yellow. A spinner has the colours shown. When the tetrahedron is rolled, the colour on its face down is recorded.



A player can choose to:

- roll the tetrahedron and spin the pointer, or
- roll the tetrahedron twice, or
- spin the pointer twice

To win, a player must make green by getting blue and yellow.

With which strategy is the player most likely to win?

Justify your answer. Play the game to check.

Show your work.



- 7.** An experiment is: rolling a regular octahedron, labelled 1 to 8, and drawing a counter from a bag that contains 4 counters: green, red, yellow, blue

The number on the octahedron and the colour of the counter are recorded.

- a) Use a regular octahedron, and a bag that contains the counters listed above.

Carry out the experiment 10 times. Record your results.

- b) Combine your results with those of 9 classmates.

- c) What is the experimental probability of each event?

i) green and a 4

ii) green or red and a 7

iii) green or yellow and an odd number

- d) Draw a tree diagram to list the possible outcomes.

- e) What is the theoretical probability of each event in part c?

- f) Compare the theoretical and experimental probabilities of the events in part c.

What do you think might happen if you carried out this experiment 1000 times? Explain.

Reflect

Which method do you prefer to find the sample space? Why?

Math Link

Your World

The Canadian Cancer Society runs a lottery every year to raise money for cancer research in Canada. One year, the chances of winning were given as the ratio 1:12. This could also be represented by the fraction $\frac{1}{12}$.