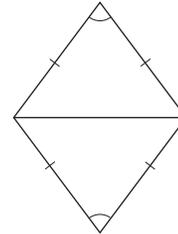## 8.3 Constructing Perpendicular Bisectors

Focus Use a variety of methods to construct perpendicular bisectors of line segments.

Recall that a rhombus has all sides equal and opposite angles equal.

Each diagonal divides the rhombus into 2 congruent isosceles triangles.
How do you know the triangles are isosceles?
How do you know the triangles are congruent?

You will investigate ways to cut line segments into 2 equal parts.

Explore

You may need rulers, protractors, tracing paper, plain paper, and Miras.
Use any methods or tools.
Draw a line segment on plain paper.
Draw a line segment perpendicular to the line segment that cuts the line segment in half.

Reflect & Share

Compare your results and methods with those of another pair of classmates.
How could you use your method to cut your classmate's line segment in half?

Connect

When you draw a line to divide a line segment into two equal parts, you **bisect** the segment.
The line you drew is a **bisector** of the segment.

When the bisector is drawn at right angles to the segment, the line is the **perpendicular bisector** of the segment.
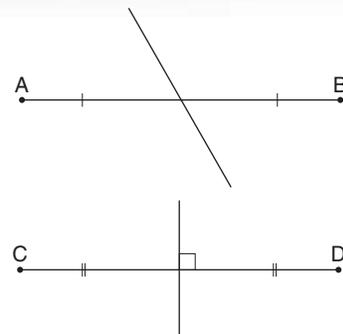
Here are 3 strategies to draw the perpendicular bisector of a given line segment.

➤ Use paper folding.
Fold the paper so that point A
lies on point B.
Crease along the fold. Open the paper.
The fold line is the perpendicular
bisector of AB.

➤ Use a Mira. Place the Mira so that the reflection
of point A lies on point B.
Draw a line segment along the edge of the Mira.

➤ Use a ruler. Place the ruler so that A is on one side
of the ruler and B is on the other.
Draw line segments along both edges of the ruler.
Repeat this step once more so that A and B are now
on opposite sides of the ruler.
Draw line segments along both edges of the ruler.
Label the points C and D where the line segments you drew intersect.
Join CD. CD is the perpendicular bisector of AB.
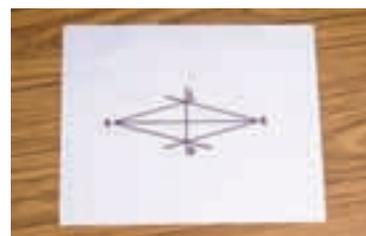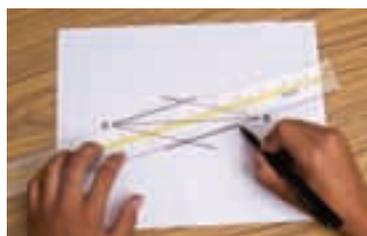
Recall that each diagonal of a rhombus is a line of symmetry.
The diagonals intersect at right angles.
The diagonals bisect each other.
So, each diagonal is the perpendicular bisector of
the other diagonal.
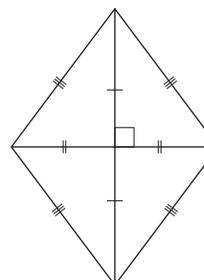
We can use these properties of a rhombus to construct
the perpendicular bisector of a line segment.
Think of the line segment as a diagonal of a rhombus.
As we construct the rhombus, we also construct the
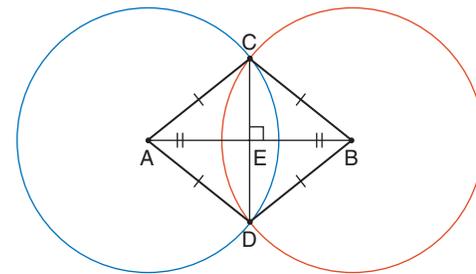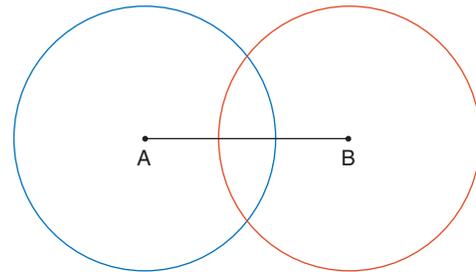perpendicular bisector of the segment.

Example

Use a ruler and compass to draw the perpendicular bisector of any line segment AB.

A Solution

Use a ruler and compass.

- Draw any line segment AB.

  A————————————B

- Set the compass so the distance between the compass and pencil points is greater than one-half the length of AB.

- Place the compass point on A.
  Draw a circle.
  Do not change the distance between the compass and pencil points.
  Place the compass point on B.
  Draw a circle to intersect the first circle you drew.

- Label the points C and D where the circles intersect.
  Join the points to form rhombus ACBD.
  Draw the diagonal CD.
  The diagonals intersect at E.
  CD is the perpendicular bisector of AB.
  That is, AE = EB and $\angle$AEC = $\angle$CEB = 90°

To check that the perpendicular bisector has been drawn correctly, measure the two parts of the segment to check they are equal, and measure the angles to check each is 90°.

> Note that any point on the perpendicular bisector of a line segment is the same distance from the endpoints of the segment.
> For example, AC = BC and AD = BD

Practice

Show all construction lines.

1. a) Draw line segment CD of length 8 cm.
   Use paper folding to draw its perpendicular bisector.
   b) Choose three different points on the bisector.
   Measure the distance to each point from C and from D.
   What do you notice?

2. a) Draw line segment EF of length 6 cm.
   Use a Mira to draw its perpendicular bisector.
   b) How do you know that you have drawn the perpendicular bisector of EF?

3. Draw line segment GH of length 4 cm.
   Use a ruler to draw its perpendicular bisector.

4. a) Draw line segment AB of length 5 cm.
   Use a ruler and compass to draw its perpendicular bisector.
   b) Choose three different points on the bisector.
   Measure the distance to each point from A and from B.
   What do you notice? Explain.

5. Find out what happens if you try to draw the perpendicular bisector of a line
   segment when the distance between the compass and pencil points is:
   a) equal to one-half the length of the segment
   b) less than one-half the length of the segment

6. **Assessment Focus** Draw line segment RS of length 7 cm.
   Use what you know about perpendicular bisectors to construct rhombus RTSU.
   How can you check that you have drawn a rhombus?

7. Look around you. Give examples of perpendicular bisectors.

8. Take It Further Draw a large △PQR.
   Construct the perpendicular bisector of each side.
   Label point C where the bisectors meet.
   Draw the circle with centre C and radius CP.

> "Circum" is Latin for "around."
> So, the *circumcircle* is the circle
> that goes around a triangle.

9. Take It Further
   a) How could you use the construction in question 8 to draw
   a circle through any 3 points that do not lie on a line?
   b) Mark 3 points as far apart as possible. Draw a circle
   through the points. Describe your construction.

> The point at which the
> perpendicular bisectors
> of the sides of a triangle
> intersect is called the
> *circumcentre*.

Reflect

How many bisectors can a line segment have?
How many perpendicular bisectors can a line segment have?
Draw a diagram to illustrate each answer.