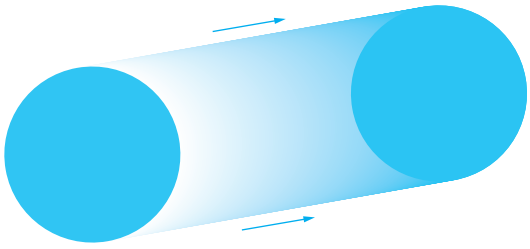
4.8 Volume of a Right Cylinder

Focus Develop and use a formula to find the volume of a right cylinder.

Here is a way to visualize a right cylinder.
A circle is translated through the air so that the circle is always parallel to its original position.



How does this relate to the triangular prism in Lesson 4.6, page 202?

Investigate

Work with a partner.
You will need 2 identical rectangular sheets of construction paper, rice, and tape.

- Roll one sheet of paper lengthwise to create a tube.
  Tape the edges together.
  Repeat with the second sheet of paper.
  This time roll the paper widthwise.
- Predict which tube has the greater volume.
  Use rice to check your prediction.
  Do the results match your prediction? Explain.
- Calculate the volume of the taller tube.
  How did you use the diameter and radius in your calculations?
  How did you use π?

Reflect & Share

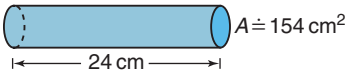
Share your strategy for calculating the volume with another pair of classmates.
Work together to write a formula for the volume of a right cylinder.
Use any of diameter, radius, height, and π in your formula.
Use your formula to find the volume of the shorter tube.

Connect

The volume of a right prism is: base area × height
We can use this formula to find the volume of a right cylinder.

Example 1

The area of the base of a cylinder is about 154 cm^2.
The height of the cylinder is 24 cm.
Find the volume of the cylinder.



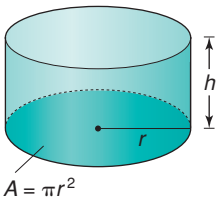
A Solution

Volume of a cylinder = base area × height
$\doteq 154 \times 24$
$= 3696$

The volume of the cylinder is about 3696 cm^3.

We can write an algebraic formula for the volume.
The base of a cylinder is a circle with radius r.
The area of a circle is: $A = \pi r^2$
Let the height of the cylinder be h.

So, the volume of a cylinder is: V = base area × height
= area of circle × height
$= \pi r^2 \times h$
$= \pi r^2 h$



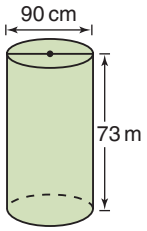
So, a formula for the volume of a cylinder is $V = \pi r^2 h$,
where r is the radius of its base, and h its height.

Example 2

In 2002, nine Pennsylvania miners were trapped in a flooded coal mine. Rescue workers drilled a hole about 90 cm wide and 73 m deep into the ground to make an escape shaft. The soil from the hole was removed and piled on the ground.
What volume of soil did the rescue workers remove? Give your answer to the nearest cubic metre.

A Solution

The hole is shaped like a cylinder.
Draw a picture. Label the cylinder.



The radius of the base is: $\frac{90 \text{ cm}}{2} = 45 \text{ cm} = 0.45 \text{ m}$
The height of the cylinder is 73 m.

Use the formula for the volume of a cylinder:
$V = \pi r^2 h$
Substitute: $r = 0.45$ and $h = 73$
$V = \pi(0.45)^2 \times 73$ Use a calculator.
$\doteq 46.44$

Both dimensions must have the same units.

The rescue workers removed 46 m^3 of soil, to the nearest cubic metre.

Discuss the ideas

1. A student measured a can of beans. The height was 10.5 cm. The diameter was 7.4 cm.
   The student calculated the volume to be about 452 cm^3.
   The label on the can shows the capacity as 398 mL.
   How is this possible?
2. Why was the base radius in *Example 2* converted from centimetres to metres?
   What would the volume be if the height was converted to centimetres?
3. In *Example 2*, why do you think the volume was asked for in cubic metres?

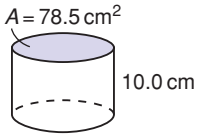
Practice

Check

Give each volume to the nearest cubic unit.

4. The base area and height of each cylinder are given to one decimal place. Calculate the volume of each cylinder.

a)



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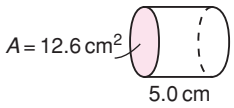
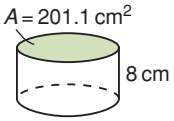
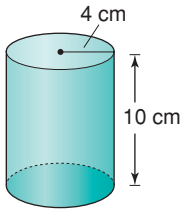


c)



5. Calculate the volume of each cylinder.

a)



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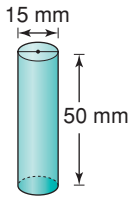

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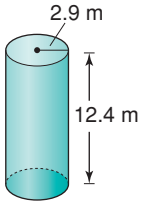


6. A candle mould is cylindrical. Its radius is 5 cm and its height is 20 cm. What volume of wax will fit in the mould?

Apply

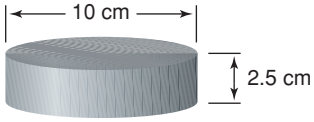
7. Find a right cylinder in the classroom.

a) Measure its height and diameter.

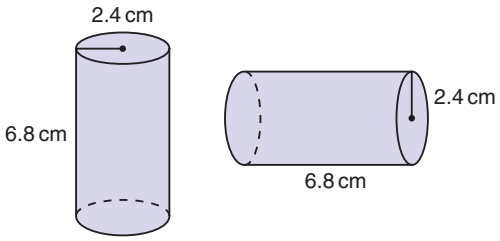
b) Calculate its base area.

c) Calculate its volume.

8. A hockey puck is a solid piece of rubber with the dimensions shown. How much rubber is used to make a hockey puck?



9. How do the volumes of these cylinders compare? How can you tell without calculating each volume?



10. Kari has 125 mL of water. She wants to pour it into one of these cylindrical bottles. Which bottle will hold all the water? How do you know?

Bottle A: $d = 7$ cm, $h = 3$ cm

Bottle B: $r = 2$ cm, $h = 6$ cm

Bottle C: $r = 3.5$ cm, $h = 7$ cm

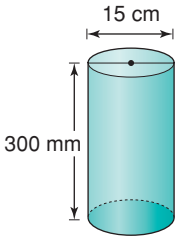
Bottle D: $d = 3$ cm, $h = 4$ cm

11. **Assessment Focus** Frozen apple juice comes in cylindrical cans. A can is 12 cm high with radius 3.5 cm.

a) What is the capacity of the can?

b) What happens to the capacity of the can if the dimensions of the radius and height are switched? Why does this happen?

12. A core sample of soil is cylindrical. The length of the core is 300 mm. Its diameter is 15 cm. Calculate the volume of soil.



13. Carol and Tom are drilling a well for water at their cottage in Lac La Hache, B.C. The drill is about 15 cm wide. Carol and Tom found water at a depth of 25 m. About how much soil did they remove before they found water?

14. A farmer has 3 cylindrical containers to hold feed. Each container has radius 91 cm and height 122 cm. What is the total volume of the three containers? How did you find out?

15. Orange juice concentrate is poured into cylindrical cans with diameter 7 cm and height 12 cm. A space of 1.5 cm is left at the top of the can to allow for expansion when the concentrate freezes. What volume of concentrate is poured into each can?

16. Take It Further Which right cylinder do you think has the greater volume?

- a cylinder with radius 1 m and height 2 m, or
- a cylinder with radius 2 m and height 1 m

How can you find out without using a calculator? Explain.

17. Take It Further

A concrete column in a parkade is cylindrical. The column is 10 m high with diameter 3.5 m.

a) What is the volume of concrete in one column?

b) There are 127 columns in the parkade. What is the total volume of concrete?

c) Suppose the concrete in part a is made into a cube. What would the dimensions of the cube be?

18. Take It Further A study shows that consumers think the diameter of a large can of coffee is too wide. The study suggests that a narrower can would increase sales. The original can has diameter 20 cm and height 18 cm. Suppose the diameter of the can is decreased by 20% without changing the volume. What is the height of the new can?

Reflect

How did your knowledge of circles help you in this lesson?