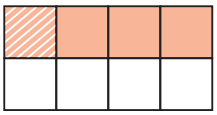
3.3 Multiplying Fractions

Focus Develop an algorithm to multiply fractions.

Which multiplication equation does this diagram represent?
How do you know?



Investigate

Work with a partner.
Use an area model to find each product.

- $\frac{2}{3} \times \frac{4}{5}$
- $\frac{1}{2} \times \frac{3}{8}$
- $\frac{3}{5} \times \frac{4}{7}$
- $\frac{2}{5} \times \frac{3}{8}$

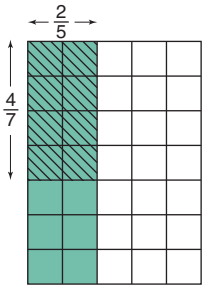
Write the multiplication equations in a table.
Look at the table.
What patterns do you notice?
How can you use patterns to multiply $\frac{2}{3} \times \frac{4}{5}$?
Use your patterns to calculate $\frac{7}{8} \times \frac{3}{10}$.
Use an area model to check your product.

Reflect & Share

Compare your strategies with those of another pair of classmates.
How does your strategy work?
Does your strategy work with $\frac{2}{5} \times \frac{3}{4}$?
Do you think your strategy will work with all fractions? Explain.

Connect

Here is an area model to show: $\frac{4}{7} \times \frac{2}{5} = \frac{8}{35}$



The product of the numerators is:
$4 \times 2 = 8$
The product of the denominators is:
$7 \times 5 = 35$
That is, $\frac{4}{7} \times \frac{2}{5} = \frac{4 \times 2}{7 \times 5}$
$= \frac{8}{35}$

Check if there are common factors in the numerator and denominator.

So, to multiply two fractions, multiply the numerators and multiply the denominators.

We can use this method to multiply proper fractions and improper fractions.

Example 1

Multiply. Estimate to check the product is reasonable.
$\frac{7}{5} \times \frac{8}{3}$

A Solution

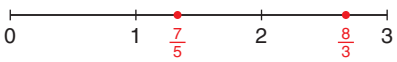
$\frac{7}{5} \times \frac{8}{3}$

There are no common factors in the numerators and denominators.
So, $\frac{7}{5} \times \frac{8}{3} = \frac{7 \times 8}{5 \times 3}$
$= \frac{56}{15}$
$= \frac{45}{15} + \frac{11}{15}$
$= 3 + \frac{11}{15}$, or $3\frac{11}{15}$

Recall that factors are the numbers that are multiplied to get a product; for example, 2 and 5 are factors of 10 because $2 \times 5 = 10$.

45 is the multiple of 15 that is closest to 56, and less than 56.

Estimate to check.
$\frac{7}{5}$ is between 1 and 2, but closer to 1.
$\frac{8}{3}$ is between 2 and 3, but closer to 3.
So, the product is about $1 \times 3 = 3$.
Since $3\frac{11}{15}$ is close to 3, the product is reasonable.



Example 2

Three-eighths of the animals in a pet store are fish.
Two-fifteenths of the fish are tropical fish.
What fraction of the animals in the pet store are tropical fish?
Use benchmarks to check the solution is reasonable.

A Solution

Since $\frac{3}{8}$ of the animals are fish and $\frac{2}{15}$ of the fish are tropical fish,
then the fraction of animals that are tropical fish is $\frac{2}{15}$ of $\frac{3}{8}$, or $\frac{2}{15} \times \frac{3}{8}$.

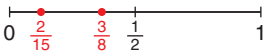
$\frac{2}{15} \times \frac{3}{8} = \frac{2 \times 3}{15 \times 8}$ Multiply the numerators and multiply the denominators.

$= \frac{6}{120}$ Simplify. Divide by the common factor, 6.

$= \frac{6 \div 6}{120 \div 6}$

$= \frac{1}{20}$

Estimate to check.
$\frac{2}{15}$ is close to 0.
$\frac{3}{8}$ is about $\frac{1}{2}$.
So, $\frac{2}{15} \times \frac{3}{8}$ is close to 0.



Since $\frac{1}{20}$ is close to 0, the product is reasonable.
One-twentieth of the animals in the pet store are tropical fish.

Example 2
Another Solution

Here is another way to calculate.
$\frac{2}{15} \times \frac{3}{8} = \frac{2 \times 3}{15 \times 8}$
Notice that the numerator and denominator have common factors 2 and 3.
To simplify first, divide the numerator and denominator by these factors.

$\frac{2}{15} \times \frac{3}{8} = \frac{\cancel{2}^{1} \times \cancel{3}^{1}}{\cancel{15}_{5} \times \cancel{8}_{4}}$

$= \frac{1 \times 1}{5 \times 4}$

$= \frac{1}{20}$

Dividing a fraction by a common factor of the numerator and denominator produces an equivalent fraction.

$2 \div 2 = 1$ $3 \div 3 = 1$
$15 \div 3 = 5$ $8 \div 2 = 4$

One-twentieth of the animals in the pet store are tropical fish.

Example 2
Another Solution

$\frac{2}{15} \times \frac{3}{8} = \frac{2 \times 3}{15 \times 8}$

The numerator and denominator have common factors 2 and 3.
Write the denominator to show the common factors.

$\frac{2}{15} \times \frac{3}{8} = \frac{2 \times 3}{3 \times 5 \times 2 \times 4}$

$= \frac{2}{2} \times \frac{3}{3} \times \frac{1}{5 \times 4}$

$= 1 \times 1 \times \frac{1}{20}$

$= \frac{1}{20}$

Rewrite making fractions that equal 1.

When multiplying by 1, the value of the fraction does not change.

One-twentieth of the animals in the pet store are tropical fish.

Discuss the ideas

1. Why is it important to estimate to check the product?
2. Look at the different solutions to *Example 2*. Why is it often helpful to simplify the fractions before multiplying?
3. How do you recognize when fractions can be simplified before you multiply them?

Practice

Check

4. Find the common factors of each pair of numbers.
   a) 4, 12 b) 14, 21 c) 8, 16
   d) 6, 9 e) 10, 15 f) 18, 24

5. Multiply: $\frac{5}{6} \times \frac{3}{20}$
   a) Multiply. Simplify first.
   b) Use benchmarks to estimate the product.
   c) Is the product reasonable? How do you know?

6. In a First Nations school, five-eighths of the Grade 8 students play the drums. Of these students, three-tenths also play the native flute. What fraction of the Grade 8 students play both the drums and the native flute? Estimate to check the solution is reasonable.

Apply

7. Multiply. Simplify before multiplying. Use benchmarks to estimate to check the product is reasonable.

a) $\frac{3}{4} \times \frac{8}{5}$ **b)** $\frac{1}{3} \times \frac{9}{10}$ **c)** $\frac{7}{5} \times \frac{15}{21}$

d) $\frac{5}{9} \times \frac{3}{5}$ **e)** $\frac{2}{9} \times \frac{15}{4}$ **f)** $\frac{7}{3} \times \frac{9}{14}$

8. Multiply. Use benchmarks to estimate to check the product is reasonable.

a) $\frac{3}{5} \times \frac{2}{3}$ **b)** $\frac{1}{2} \times \frac{5}{10}$ **c)** $\frac{1}{6} \times \frac{1}{4}$

d) $\frac{13}{8} \times \frac{3}{2}$ **e)** $\frac{5}{4} \times \frac{11}{10}$ **f)** $\frac{7}{3} \times \frac{7}{8}$

Which of these questions could have been solved using mental math? Justify your choice.

9. Solve each problem. Estimate to check the solution is reasonable.

a) Josten took $\frac{3}{8}$ of his savings on a shopping trip. He used $\frac{1}{4}$ of the money to buy a new coat. What fraction of his savings did Josten spend on the coat?

b) Gervais ate $\frac{1}{3}$ of a baguette with his dinner. Chantel ate $\frac{1}{4}$ of the leftover baguette as an evening snack. What fraction of the baguette did Chantel eat as a snack?

10. Write a story problem that can be represented by the expression $\frac{7}{8} \times \frac{1}{2}$.
Solve your problem.
Trade problems with a classmate.
Solve your classmate's problem.
Check to see that your solutions are the same.

11. Eeva spent $\frac{5}{6}$ of $\frac{3}{4}$ of her total allowance on a hair crimper. What fraction of her total allowance did Eeva have left?

12. **a)** Find each product.

i) $\frac{3}{4} \times \frac{4}{3}$ **ii)** $\frac{1}{5} \times \frac{5}{1}$

iii) $\frac{7}{2} \times \frac{2}{7}$ **iv)** $\frac{5}{6} \times \frac{6}{5}$

b) What do you notice about the products in part a? Write 3 more pairs of fractions that have the same product. What can you say about the product of a fraction and its reciprocal?

$\frac{11}{12}$ and $\frac{12}{11}$ are reciprocals.

13. **Assessment Focus** In question 12, each product is 1.

a) Write a pair of fractions that have each product.

i) 2 **ii)** 3 **iii)** 4 **iv)** 5

b) Write a pair of fractions that have the product 1. Change only one numerator or denominator each time to write a pair of fractions that have each product.

i) 2 **ii)** 3 **iii)** 4 **iv)** 5

c) How can you write a pair of fractions that have the product 10? Show your work.

14. The sum of two fractions is $\frac{7}{12}$. The product of the same two fractions is $\frac{1}{12}$. What are the two fractions? Describe the strategy you used.

15. Multiply. Estimate to check the product is reasonable.

a) $\frac{33}{40} \times \frac{15}{55}$

b) $\frac{26}{39} \times \frac{9}{13}$

c) $\frac{51}{64} \times \frac{8}{17}$

d) $\frac{76}{91} \times \frac{7}{19}$

16. **a)** Multiply $\frac{24}{25} \times \frac{85}{96}$ using each strategy below.

i) Simplify before multiplying.

ii) Multiply first, then simplify.

b) Which strategy in part a did you find easier? Justify your choice.

17. The product of 2 fractions is $\frac{3}{4}$.
What might the fractions be?
How many pairs of fractions could have a product of $\frac{3}{4}$? How do you know?

18. **Take It Further** Keydon baked a wild blueberry upside-down cobbler.

Shawnie ate $\frac{1}{6}$ of the cobbler.

Iris ate $\frac{1}{5}$ of what was left.

Chan ate $\frac{1}{4}$ of what was left after that.

Cami ate $\frac{1}{3}$ of what was left after that.

Demi ate $\frac{1}{2}$ of what was left after that.
How much of the original cobbler remained?

19. **Take It Further** The product of two fractions is $\frac{2}{3}$. One fraction is $\frac{3}{5}$.
What is the other fraction? How do you know?

20. **Take It Further** Eddie used the expression $\frac{4}{7} \times \frac{3}{5}$ to solve a word problem. Which of these word problems better fits the expression? How do you know? Solve the problem.

a) $\frac{4}{7}$ of the Grade 8 students voted to have Spirit Day. $\frac{3}{5}$ of those students wanted Spirit Day to be on the first day of classes. What fraction of the Grade 8 students wanted Spirit Day to be on the first day of classes?

b) $\frac{3}{5}$ of the Grade 7 students voted to have a school dance. $\frac{4}{7}$ of those students wanted the dance to be on the day before Spring Break.
What fraction of the Grade 7 students wanted the dance to be on the day before Spring Break?

21. **Take It Further**
Find each square root.
Explain the strategy you used.

a) $\sqrt{\frac{4}{9}}$ **b)** $\sqrt{\frac{16}{25}}$

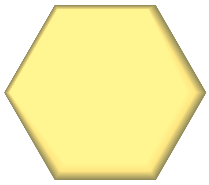
c) $\sqrt{\frac{36}{81}}$ **d)** $\sqrt{\frac{49}{169}}$

Reflect

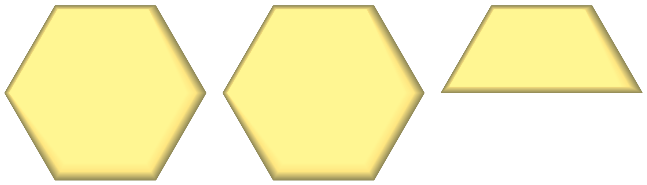
When we multiply 2 whole numbers, the product is always greater than either factor.
Is this always true when we multiply 2 fractions?
Use examples and diagrams to explain your answer.

3.4 Multiplying Mixed Numbers

Focus Apply knowledge of multiplying fractions to multiply mixed numbers.

Suppose  = 1.

How can you write the fraction representing



in 2 ways?

Investigate

Work with a partner.

During the salmon drift,
volunteers collect catch information from fisherpeople.
Akecheta volunteered for $3\frac{1}{2}$ h.

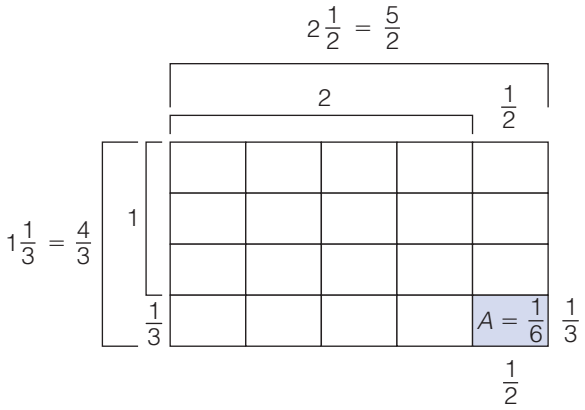
Onida volunteered for $\frac{2}{3}$ of the time
that Akecheta volunteered.
For how long did Onida volunteer?
How can you find out?
Show your work.
Use models or diagrams to justify your strategy.

Reflect & Share

Compare your strategy with that of another pair of classmates.
Do you think your strategy will work with all mixed numbers?
Test it with $\frac{3}{4} \times 2\frac{1}{3}$.

Connect

Here is an area model to show: $2\frac{1}{2} \times 1\frac{1}{3}$



Write each mixed number as an improper fraction.

$2\frac{1}{2} = 2 + \frac{1}{2} = \frac{4}{2} + \frac{1}{2} = \frac{5}{2}$

$1\frac{1}{3} = 1 + \frac{1}{3} = \frac{3}{3} + \frac{1}{3} = \frac{4}{3}$

Each small rectangle has area: $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

There are 5×4, or 20 small rectangles.

So, the area is: $20 \times \frac{1}{6} = \frac{20}{6}$

$= \frac{10}{3}$

$= 3\frac{1}{3}$

After we write the mixed numbers as improper fractions, we can multiply the same way we multiplied proper fractions.

$2\frac{1}{2} \times 1\frac{1}{3} = \frac{5}{2} \times \frac{4}{3}$

$= \frac{5 \times 4}{2 \times 3}$

$= \frac{20}{6}$

$= \frac{20 \div 2}{6 \div 2}$

$= \frac{10}{3}$

$= 3\frac{1}{3}$

This is the same product as when we used the area model.
We can also use a rectangle model to multiply two mixed numbers.

Example 1

Multiply: $2\frac{1}{2} \times 1\frac{1}{3}$

A Solution

Use a rectangle model.

$2\frac{1}{2} \times 1\frac{1}{3} = (2 \times 1) + (\frac{1}{2} \times 1) + (2 \times \frac{1}{3}) + (\frac{1}{2} \times \frac{1}{3})$

$= 2 + \frac{1}{2} + \frac{2}{3} + \frac{1}{6}$

$= 2 + \frac{3}{6} + \frac{4}{6} + \frac{1}{6}$

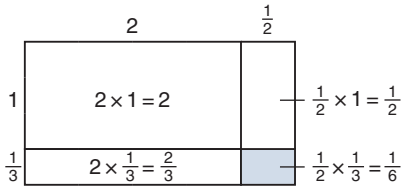
$= 2 + \frac{8}{6}$

$= 2 + \frac{6}{6} + \frac{2}{6}$

$= 2 + 1 + \frac{2}{6}$

$= 3\frac{2}{6}$, or $3\frac{1}{3}$

Add. Use common denominators.



Remember to write the product in simplest form.

Example 2

Multiply. Estimate to check the product is reasonable.

$2\frac{1}{4} \times 3\frac{2}{5}$

A Solution

$2\frac{1}{4} \times 3\frac{2}{5}$

$2\frac{1}{4} = 2 + \frac{1}{4}$

$= \frac{8}{4} + \frac{1}{4}$

$= \frac{9}{4}$

and

$3\frac{2}{5} = 3 + \frac{2}{5}$

$= \frac{15}{5} + \frac{2}{5}$

$= \frac{17}{5}$

So, $2\frac{1}{4} \times 3\frac{2}{5} = \frac{9}{4} \times \frac{17}{5}$

$= \frac{153}{20}$

$= \frac{140}{20} + \frac{13}{20}$

$= 7\frac{13}{20}$

Estimate to check.

$2\frac{1}{4}$ is between 2 and 3, but closer to 2.

$3\frac{2}{5}$ is between 3 and 4, but closer to 3.

So, the product is about $2 \times 3 = 6$.

Since $7\frac{13}{20}$ is close to 6, the product is reasonable.

Example 3

Multiply. Estimate to check the product is reasonable.

$3\frac{3}{8} \times 4\frac{2}{3}$

A Solution

$3\frac{3}{8} \times 4\frac{2}{3}$

$3\frac{3}{8} = 3 + \frac{3}{8}$ and $4\frac{2}{3} = 4 + \frac{2}{3}$

$= \frac{24}{8} + \frac{3}{8}$ $= \frac{12}{3} + \frac{2}{3}$

$= \frac{27}{8}$ $= \frac{14}{3}$

So, $3\frac{3}{8} \times 4\frac{2}{3} = {}^{9}\!\cancel{27}_{4}\cancel{8} \times \cancel{14}^{7}\cancel{3}_{1}$

$= \frac{9 \times 7}{4 \times 1}$

$= \frac{63}{4}$

$= \frac{60}{4} + \frac{3}{4}$

$= 15\frac{3}{4}$

Divide by common factors.

27 ÷ 3 = 9	14 ÷ 2 = 7
8 ÷ 2 = 4	3 ÷ 3 = 1

Estimate to check.

$3\frac{3}{8}$ is between 3 and 4, but closer to 3.

$4\frac{2}{3}$ is between 4 and 5, but closer to 5.

So, the product is about $3 \times 5 = 15$.

Since $15\frac{3}{4}$ is close to 15, the product is reasonable.

Discuss the ideas

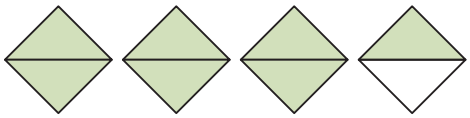
1. What is the difference between a proper fraction and an improper fraction?
2. How is multiplying two mixed numbers like multiplying two fractions?
3. How is the rectangle model useful when you multiply 2 mixed numbers?

Practice

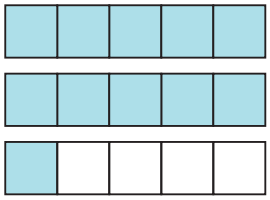
Check

4. Write the mixed number and improper fraction represented by each picture.

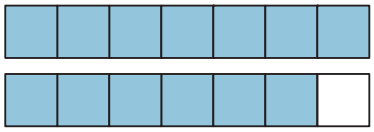
a)



b)



c)



5. Write each mixed number as an improper fraction.

a) $2\frac{3}{10}$ b) $4\frac{1}{8}$ c) $3\frac{5}{6}$

d) $1\frac{2}{3}$ e) $3\frac{2}{5}$ f) $5\frac{1}{2}$

g) $2\frac{4}{7}$ h) $3\frac{5}{9}$ i) $6\frac{2}{3}$

6. Write each improper fraction as a mixed number.

a) $\frac{11}{3}$ b) $\frac{15}{4}$ c) $\frac{21}{5}$

d) $\frac{11}{8}$ e) $\frac{19}{6}$ f) $\frac{31}{7}$

g) $\frac{11}{2}$ h) $\frac{43}{10}$ i) $\frac{37}{8}$

7. Use estimation. Which number is each product closer to?

a) $2\frac{1}{8} \times 3\frac{3}{4}$ 6 or 8

b) $3\frac{5}{9} \times 1\frac{5}{6}$ 6 or 8

c) $7\frac{3}{8} \times 2\frac{4}{5}$ 21 or 24

d) $4\frac{7}{9} \times 3\frac{5}{12}$ 15 or 20

8. Multiply: $3\frac{3}{5} \times 2\frac{2}{9}$

a) Estimate the product.

b) Write each mixed number as an improper fraction.

c) Multiply the improper fractions. Simplify first.

d) Is the product reasonable? How do you know?

Apply

9. Multiply. Estimate to check the product is reasonable.

a) $3 \times 2\frac{1}{4}$ b) $4 \times 2\frac{1}{8}$

c) $1\frac{2}{3} \times 2$ d) $3\frac{1}{5} \times 3$

10. Use an area model to find each product.

a) $1\frac{1}{2} \times 1\frac{1}{3}$ b) $2\frac{3}{4} \times 2\frac{2}{3}$

c) $1\frac{1}{5} \times 3\frac{1}{3}$ d) $1\frac{1}{2} \times 2\frac{2}{5}$

11. Use improper fractions to find each product. Estimate to check the product is reasonable.

a) $1\frac{7}{8} \times 2\frac{2}{3}$ b) $4\frac{1}{6} \times 3\frac{2}{5}$

c) $2\frac{3}{7} \times 1\frac{5}{9}$ d) $3\frac{1}{2} \times 2\frac{2}{7}$

e) $2\frac{1}{4} \times 2\frac{2}{3}$ f) $1\frac{4}{5} \times 2\frac{1}{3}$

12. Multiply. Estimate to check the product is reasonable.

a) $1\frac{3}{4} \times 2\frac{1}{2}$ b) $3\frac{2}{3} \times 2\frac{1}{5}$

c) $4\frac{3}{8} \times 1\frac{1}{4}$ d) $3\frac{3}{4} \times 3\frac{3}{4}$

e) $4\frac{3}{10} \times \frac{4}{5}$ f) $\frac{7}{8} \times 2\frac{3}{5}$

13. A restaurant in Richmond, BC, lists the prices on its menu in fractions of a dollar. Three friends have lunch at the restaurant. Each of 3 friends orders a veggie mushroom cheddar burger for $11\frac{3}{4}$, with a glass of water to drink.

a) What was the total bill before taxes, in fractions of a dollar?

b) What was the total bill before taxes, in dollars and cents?

14. During the school year, the swim team practises $2\frac{3}{4}$ h per week. During the summer, the weekly practice time is increased to $2\frac{1}{3}$ times the school-year practice time. How many hours per week does the team practise during the summer?

15. Write a story problem that can be represented by the expression $3\frac{1}{2} \times 2\frac{1}{8}$. Solve your problem. Trade problems with a classmate. Solve your classmate's problem. Check to see that your solutions are the same.

16. In a baseball game, the starting pitcher for the home team pitched $4\frac{2}{3}$ innings. The starting pitcher for the visiting team pitched $1\frac{1}{2}$ times as many innings. How many innings did the visiting team's pitcher pitch?

17. **Assessment Focus** Students baked cookies for a charity bake sale. Elsa baked $2\frac{1}{2}$ dozen cookies. Layton baked $2\frac{1}{6}$ times as many cookies as Elsa. Meghan and Josh together baked $5\frac{1}{3}$ times the number of cookies that Elsa baked.

a) Estimate. About how many dozen cookies did Layton bake? About how many dozen cookies did Meghan and Josh bake altogether?

b) Calculate how many dozen cookies Layton baked.

c) Calculate how many dozen cookies Meghan and Josh baked.

d) How many dozen cookies did these 4 students bake altogether?

e) How many cookies did these 4 students bake altogether?

Show your work.

18. **Take It Further** Use estimation. Which expression below has the greatest product? The least product? How do you know?

a) $\frac{4}{3} \times \frac{8}{6}$ **b)** $2\frac{1}{8} \times 1\frac{1}{5}$

c) $1\frac{3}{8} \times \frac{9}{4}$ **d)** $\frac{7}{2} \times 2\frac{3}{10}$

19. **Take It Further** Multiply. Estimate to check the product is reasonable.

a) $2\frac{4}{9} \times 2\frac{2}{3} \times 2\frac{1}{2}$

b) $3\frac{3}{5} \times 2\frac{3}{4} \times 1\frac{1}{4}$

c) $4\frac{3}{8} \times 1\frac{1}{5} \times 2\frac{1}{4}$

Reflect

Describe 2 strategies you can use to multiply $3\frac{1}{2} \times 5\frac{1}{4}$.
Which strategy do you prefer? Why?