4.3 Area of a Parallelogram
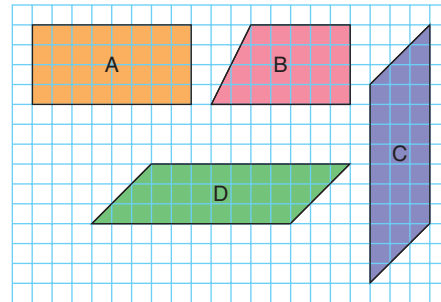
Focus Develop a formula to find the area of a parallelogram.

Which of these shapes are parallelograms?
How do you know?

How are Shapes C and D alike?
How are they different?

Explore

You will need scissors and 1-cm grid paper.

➤ Copy Parallelogram A
on grid paper.
Estimate, then find, the
area of the parallelogram.

➤ Cut out the parallelogram.
Then, cut along the broken
line segment.

➤ Arrange the two pieces to form a rectangle.
What is the area of the rectangle?
How does the area of the rectangle compare to the area
of the parallelogram?

➤ Repeat the activity for Parallelograms B and C.
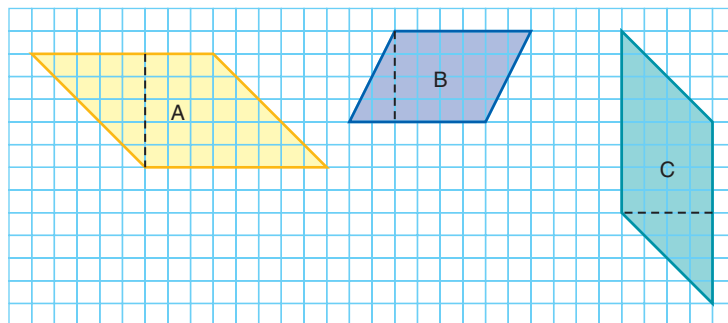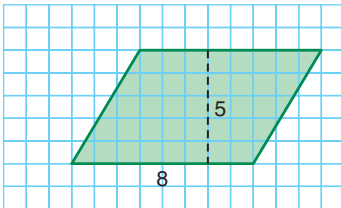
Reflect & Share

Share your work with another pair of classmates.
Can every parallelogram be changed into a rectangle
by cutting and moving one piece? Explain.
Work together to write a rule for finding the area of a parallelogram.

To estimate the area of this parallelogram, count the whole squares and the part squares that are one-half or greater.

There are:
- **33 whole squares**
- **8 part squares that are one-half or greater**

The area of this parallelogram is about 41 square units.

Any side of a parallelogram is a **base** of the parallelogram. The **height** of a parallelogram is the length of a line segment that joins parallel sides and is perpendicular to the base.
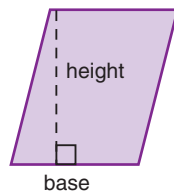
Recall that both a rectangle and a square are parallelograms.

Any parallelogram that is not a rectangle can be "cut" and rearranged to form a rectangle. Here is one way to do this.
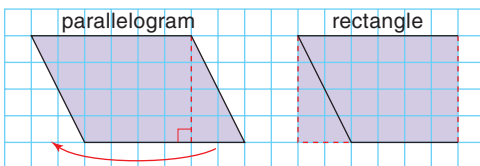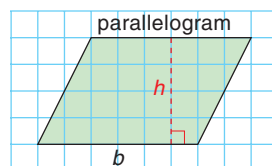
The parallelogram and the rectangle have the same area.
The area of a parallelogram is equal to the area of a rectangle with the same height and base.
To find the area of a parallelogram, multiply the base by the height.

b represents the base.
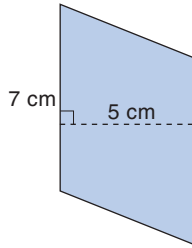*h* represents the height.
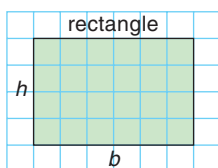
Area of rectangle:
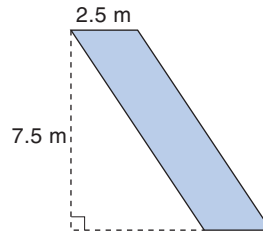$A = bh$

Area of parallelogram:
$A = bh$

Example

Calculate the area of each parallelogram.

a)

b)

> The height can be drawn outside the parallelogram.

A Solution

The area of a parallelogram is given by the formula $A = bh$.

a) $A = bh$

Substitute: $b = 7$ and $h = 5$

$A = 7 \times 5$

$= 35$

The area of the parallelogram is 35 cm².

b) $A = bh$

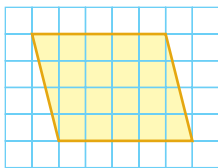Substitute: $b = 2.5$ and $h = 7.5$

$A = 2.5 \times 7.5$
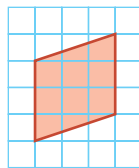
$= 18.75$

The area of the parallelogram is 18.75 m².

Practice

1. i) Copy each parallelogram on 1-cm grid paper.
   ii) Show how the parallelogram can be rearranged to form a rectangle.
   iii) Estimate, then find, the area of each parallelogram.
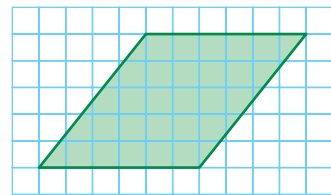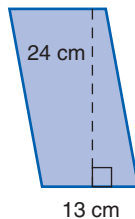
   a)

   b)

   c)

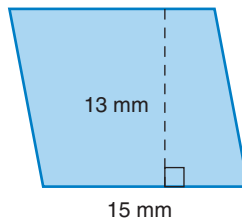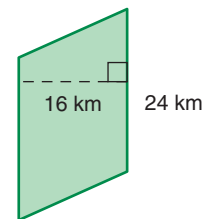2. Find the area of each parallelogram.

   a)

   b)

   c)

3. a) On 1-cm grid paper, draw 3 different parallelograms with base 3 cm and height 7 cm.
   b) Find the area of each parallelogram you drew in part a. What do you notice?

4. Repeat question 3. This time, you choose the base and height.
Are your conclusions the same as in question 3? Why or why not?

5. Copy this parallelogram on 1-cm grid paper.
   a) Show how this parallelogram could be rearranged
   to form a rectangle.
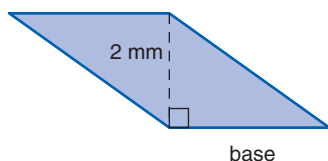   **b)** Find the area of the parallelogram.

2 cm

5 cm
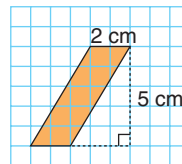
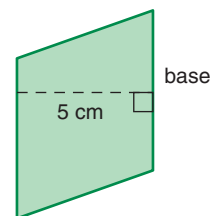**6.** Use the given area to find the base or the height of each parallelogram.
   a) Area = 60 m²   **b)** Area = 6 mm²   **c)** Area = 30 cm²

height

12 m

2 mm

base

base

5 cm
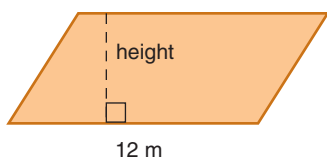
7. On 1-cm grid paper, draw as many different
parallelograms as you can with each area.
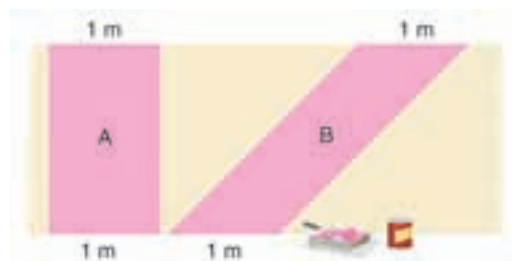   **a)** 10 cm²   **b)** 18 cm²   **c)** 28 cm²

4 cm

5 cm

8. A student says the area of this parallelogram is 20 cm².
Explain the student's error.

4 cm
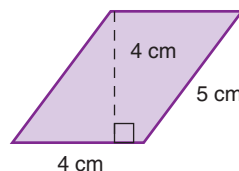
9. **Assessment Focus** Sasha is buying paint
for a design on a wall. Here is part of the design.
Sasha says Shape B will need
more paint than Shape A.
Do you agree? Why or why not?

1 m   1 m

A   B

1 m   1 m

10. **Take It Further** A restaurant owner built a patio
in front of his store to attract more customers.
   a) What is the area of the patio?
   b) What is the total area of the patio and gardens?
   c) How can you find the area of the gardens?
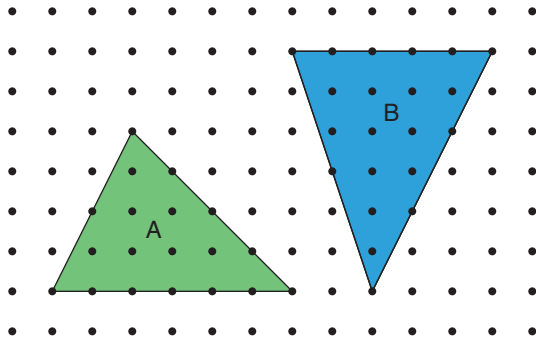   Show your work.

14.4 m   5.6 m

Garden

Patio

6.6 m

Garden

5.6 m   14.4 m

Reflect

How can you use what you know about rectangles
to help you find the area of a parallelogram?
Use an example to explain.

4.4 Area of a Triangle

Focus Develop and use a formula to find the area of a triangle.

Explore

You will need a geoboard, geobands, and dot paper.

➤ Make Triangle A on a geoboard.
Add a second geoband to Triangle A to make
a parallelogram with the same base and height.
This is called a *related* parallelogram.
Make as many different parallelograms as you can.
How does the area of the parallelogram
compare to the area of Triangle A each time?
Record your work on dot paper.

➤ Repeat the activity with Triangle B.

➤ What is the area of Triangle A? Triangle B?
What strategy did you use to find the areas?

Reflect & Share

Share the different parallelograms you made with another pair of classmates.
Discuss the strategies you used to find the area of each triangle.
How did you use what you know about a parallelogram
to find the area of a triangle?
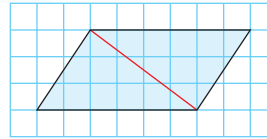Work together to write a rule for finding the area of a triangle.

When we draw a diagonal in a parallelogram,
we make two congruent triangles.
Congruent triangles have the same area.
The area of the two congruent triangles
is equal to the area of the parallelogram
that contains them.
So, the area of one triangle is $\frac{1}{2}$ the area
of the parallelogram.

To find the area of this triangle:

Complete a parallelogram on
one side of the triangle.
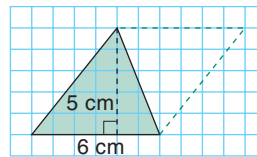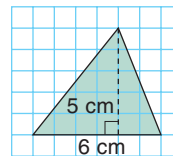The area of the parallelogram is:
$A = $ base $\times$ height, or $A = bh$
So, $A = 6 \times 5$
$ = 30$
The area of the parallelogram is 30 cm².
So, the area of the triangle is: $\frac{1}{2}$ of 30 cm² $= 15$ cm²

We can write a formula for the area of a triangle.
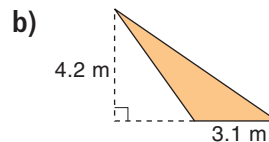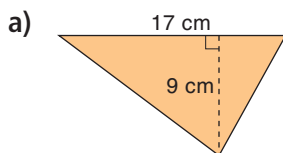The area of a parallelogram is:
$A = $ base $\times$ height
So, the area of a triangle is:
$A = $ one-half of base $\times$ height
$A = bh \div 2$, which can be written as $A = \frac{bh}{2}$

Example

Find the area of each triangle.

a)   17 cm

9 cm

b)   4.2 m

3.1 m

> For an obtuse triangle,
> the height might be drawn
> outside the triangle.

A Solution

a) $A = \frac{bh}{2}$

Substitute: $b = 17$ and $h = 9$

$A = \frac{17 \times 9}{2}$

$= \frac{153}{2}$

$= 76.5$

The area is 76.5 cm².

b) $A = \frac{bh}{2}$

Substitute: $b = 3.1$ and $h = 4.2$

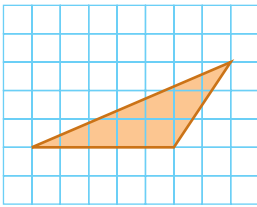$A = \frac{3.1 \times 4.2}{2}$

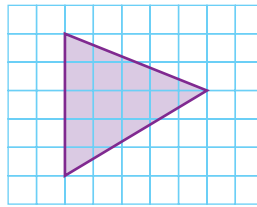$= \frac{13.02}{2}$

$= 6.51$

The area is 6.51 m².

Practice

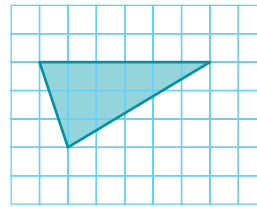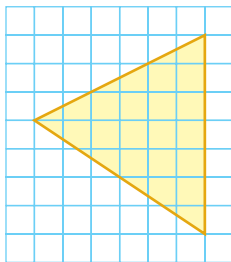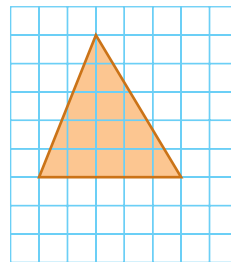**1.** Copy each triangle on 1-cm grid paper. Draw a related parallelogram.

a)

b)

c)

2. Each triangle is drawn on 1-cm grid paper.
Find the area of each triangle. Use a geoboard if you can.

a)

b)

c)

d)

e)

f)

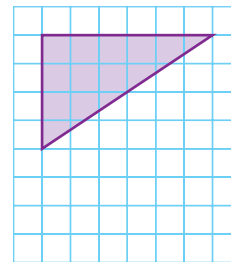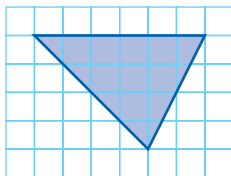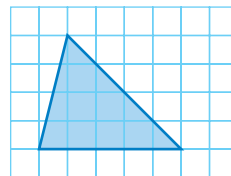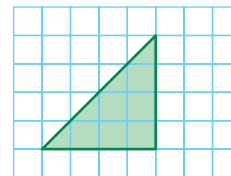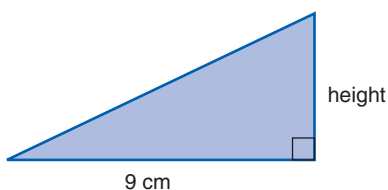**3.** Draw two right triangles on 1-cm grid paper.
   a) Record the base and the height of each triangle.
   b) What do you notice about the height of a right triangle?
   c) Find the area of each triangle you drew.

4. a) Find the area of this triangle.

   b) Use 1-cm grid paper.
How many different parallelograms can
you draw that have the same base
and the same height as this triangle?
Sketch each parallelogram.

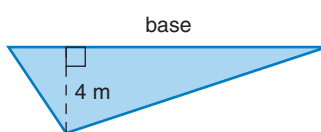   **c)** Find the area of each parallelogram.
What do you notice?
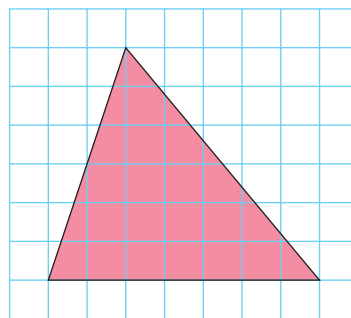
5. Use the given area to find the base or height of each triangle.
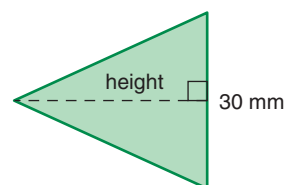How could you check your answers?

   a) Area = 18 cm²   **b)** Area = 32 m²   **c)** Area = 480 mm²

height   base   height

9 cm   4 m   30 mm

6. Use 1-cm grid paper.

   a) Draw 3 different triangles with each base and height.

     i) base: 1 cm; height: 12 cm

    ii) base: 2 cm; height: 6 cm

   iii) base: 3 cm; height: 4 cm

   b) Find the area of each triangle you drew in part a.
What do you notice?

7. On 1-cm grid paper, draw two different triangles with each area below.
Label the base and height each time.
How do you know these measures are correct?

   a) 14 cm²   **b)** 10 cm²   **c)** 8 cm²

8. a) Draw any triangle on grid paper.
What happens to the area of the triangle in each case?

     i) the base is doubled

    ii) both the height and the base are doubled

   iii) both the height and the base are tripled

   b) What could you do to the triangle you drew in part a to triple its area?
Explain why this would triple the area.

9. Assessment Focus

This triangle is made from 4 congruent triangles.
Three triangles are to be painted blue.
The fourth triangle is not to be painted.
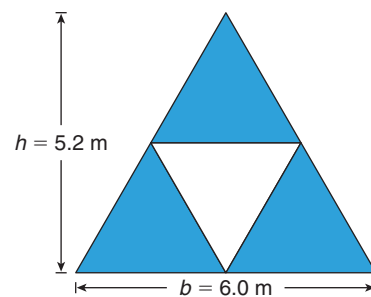
a) What is the area that is to be painted?
   Show your work.

b) The paint is sold in 1-L cans.
   One litre of paint covers 5.5 m².
   How many cans of paint are needed?
   What assumptions did you make?

$h = 5.2$ m
$b = 6.0$ m

The height is approximate.

10. Look at the diagram to the right.

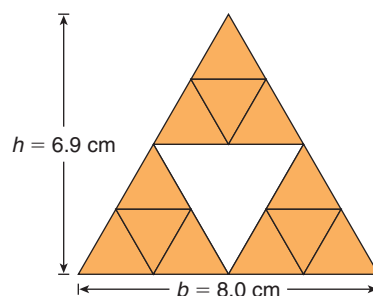a) How many triangles do you see?
b) How are the triangles related?
c) How many parallelograms do you see?
d) Find the area of the large triangle.
e) Find the area of one medium-sized triangle.
f) Find the area of one small triangle.
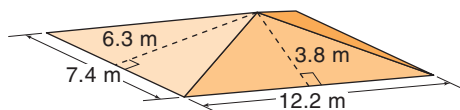g) Find the area of a parallelogram of your choice.

$h = 6.9$ cm
$b = 8.0$ cm

The height is approximate.

11. Take It Further

A local park has a pavilion to provide shelter.
The pavilion has a roof the shape of a rectangular pyramid.

6.3 m
3.8 m
7.4 m
12.2 m

a) What is the total area of all four parts of the roof?
b) One sheet of plywood is 240 cm by 120 cm.
   What is the least number of sheets of plywood
   needed to cover the roof?
   Explain how you got your answer.

Reflect

What do you know about finding the area of a triangle?