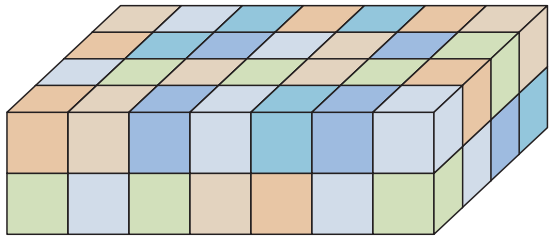
4.5 Volume of a Right Rectangular Prism

Focus Develop and use a formula to calculate the volume of a right rectangular prism.

This rectangular prism is made from 1-cm cubes.
What is the volume of the prism?



Investigate

Work with a partner.
You will need 2 empty cereal boxes, a ruler, and a calculator.

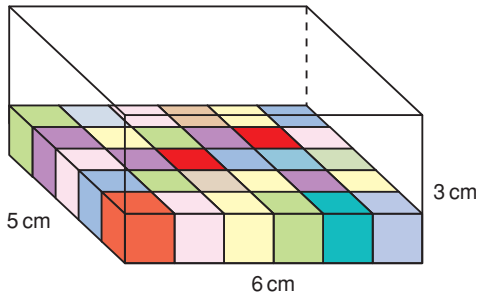
- Compare the two boxes.
  Which box do you think holds more cereal?
  Why do you think so?
- Find the volume of each box.
  Which box has the greater volume?
  How does this compare with your prediction?
- Work together to write a formula you can use to find the volume of any rectangular prism.
- Suppose you know the area of one face of a cereal box. What else do you need to know to find the volume of the box?
- Work together to write a formula for the volume of a right rectangular prism in terms of the area of its base.

Reflect & Share

Compare your formula with that of another pair of classmates.
Did you write the same formula?
If not, do both formulas work? Explain.
Compare the formulas for the volume of a right rectangular prism and the area of a rectangle. What do you notice?
How can you explain this?

Connect

This box is a right rectangular prism.
The volume of the box is the number of centimetre cubes the box holds.

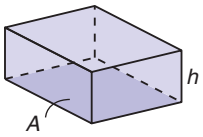


Recall that, when the dimensions are measured in centimetres (cm), the volume is measured in cubic centimetres (cm^3).

One layer of cubes will be 5 cm wide and 6 cm long.
So, 5×6, or 30 cubes fit in one layer.
The box is 3 cm high, so 3 layers can fit.
The total number of cubes is $30 \times 3 = 90$.
So, the volume of the box is 90 cm^3.

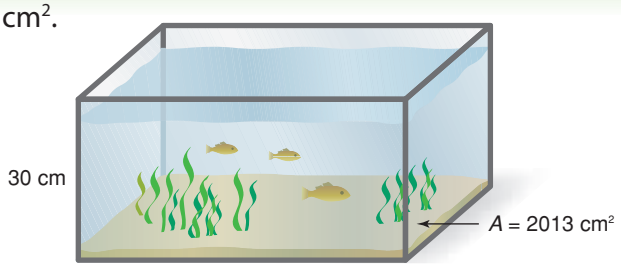
One way to write the volume of the box:
Volume = the number of cubes in one layer $\times$ the number of layers
This is the same as:
Volume = the area of the base of the box $\times$ its height
This is true for all rectangular prisms.

We can use variables to write a formula for the volume of a rectangular prism.
Let A represent the base area and h represent the height.
Then, the volume of a rectangular prism is:
$V = A \times h$ or $V = Ah$



Example 1

The area of the base of a fish tank is 2013 cm^2.
The height of the tank is 30 cm.
Find the volume of the fish tank.



A Solution

The fish tank is a right rectangular prism with base area 2013 cm^2 and height 30 cm.
Volume = base area $\times$ height
= 2013 $\times$ 30
= 60 390

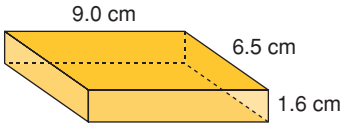
The volume of the fish tank is 60 390 cm^3.

Example 2

A deck of 54 cards fits in a box shaped like a right rectangular prism.
The box has dimensions 6.5 cm by 9.0 cm by 1.6 cm.
What is the volume of the box?
Give the answer to the nearest cubic centimetre.

A Solution

Draw a diagram.
Label each dimension.
Let the base be one rectangle with length 9.0 cm and width 6.5 cm.



$A = 9.0 \times 6.5$
$= 58.5$
The area of the base is 58.5 cm^2.
The height of the box is 1.6 cm.
Use the formula: $V = Ah$
$V = 58.5 \times 1.6$ Use a calculator.
$= 93.6$
The volume is 94 cm^3, to the nearest cubic centimetre.

Discuss the ideas

1. Suppose the rectangular prism in *Connect* holds 210 centimetre cubes. How high is the box? Assume the area of the base is unchanged.
2. When you find the volume of a right rectangular prism, does it matter which face you use as the base?
3. For *Example 2*, suggest a different way to find the volume of the box.

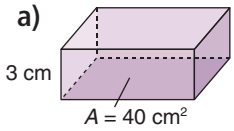
Practice

Use a calculator when it helps.

Check

4. The base area and height of each prism are given. Find the volume of each prism.

a)



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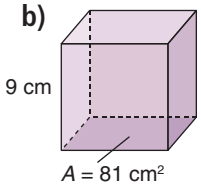

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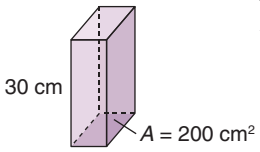
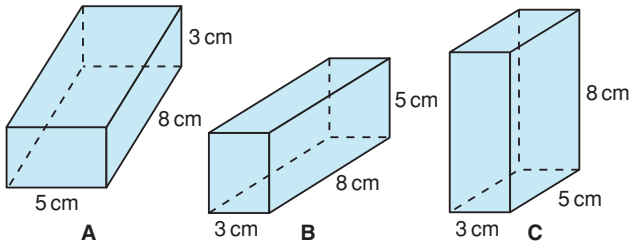


5. A box of laundry detergent has dimensions 28 cm by 16 cm by 25 cm.

a) Sketch the box.
Label each dimension.

b) What volume of detergent will fill the box?

6. **a)** Find the volume of each prism.



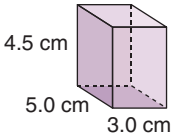
b) What do you notice about the volumes in part a?

c) Does the volume of a rectangular prism change when you place the prism on a different base? Justify your answer.

Apply

7. Find the volume of each rectangular prism.

a)



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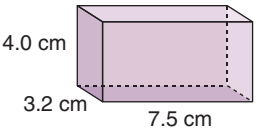

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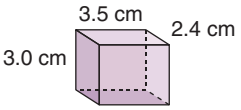


8. Find a right rectangular prism in the classroom. Measure its dimensions. Find its volume.

9. Each dogsled team that enters the Iditarod has a portable doghouse for each sled dog. Two mushers are comparing the sizes of their doghouses. Each of Rick's doghouses is 94 cm by 63 cm by 71 cm. Each of Susan's doghouses is 109 cm by 71 cm by 81 cm.

a) What is the volume of each doghouse?

b) About how many times as great as the volume of Rick's doghouse is the volume of Susan's doghouse?

10. Suppose a milk carton is 10 cm wide and 10 cm long. How tall must the carton be to hold 1 L of milk?

Recall 1 cm^3 = 1 mL.

11. Large trucks often tow trailers that are shaped like right rectangular prisms. A standard trailer is 2.74 m by 2.43 m by 6.1 m.

a) What is the greatest volume of cargo a standard trailer can hold?

b) How many trailers would it take to transport 100 m^3 of goods?
What assumptions do you make?

12. A rectangular swimming pool is to be filled with water. The pool has a uniform depth of 2 m and is surrounded by a wooden deck.
The pool is 20 m wide and 50 m long.
How much water is needed in each case?

a) The pool is filled to the level of the deck.

b) The pool is filled to within 20 cm of the level of the deck.

c) The pool is half filled.

13. **Assessment Focus**

a) Sketch all possible right rectangular prisms with volume 36 cm^3.
Label each prism with its dimensions in centimetres.
How do you know you have found all possible prisms?

b) Use the prisms you sketched.

i) Which prism has the greatest surface area?

ii) Which prism has the least surface area?
How did you find out?

14. Philip made fudge that filled a 20-cm by 21-cm by 3-cm pan.

a) What is the volume of the fudge?

b) Philip shares the fudge with his classmates.
There are 30 people in the class.
How much fudge will each person get?

c) How could Philip cut the fudge so each person gets an equal sized piece?
Sketch the cuts Philip could make.

d) What are the dimensions of each piece of fudge in part c?

15. Sketch a right rectangular prism.
Label its dimensions.
What do you think happens to the volume of the prism when:

a) its length is doubled?

b) its length and width are doubled?

c) its length, width, and height are doubled?

Investigate to find out.
Show your work.
Will the results be true for all rectangular prisms?
Why do you think so?

16. **Take It Further**
How can you double the volume of a right rectangular prism?
Does its surface area double, too?
Explain.

17. **Take It Further**
Students in a Grade 8 class are filling shoeboxes with toys for children in other countries. A shoebox measures 30 cm by 18 cm by 16 cm.

a) Find the volume of a shoebox.

b) The students fill 24 shoeboxes. Eight shoeboxes are packed into a larger box. What could the dimensions of this larger box be?

c) What are the most likely dimensions of the larger box? Justify your choice.

18. **Take It Further**

a) Sketch 3 different right rectangular prisms with volume 24 cm^3.

b) Which prism has the greatest surface area?
Which prism has the least surface area?

c) Sketch a prism with a greater surface area but the same volume.
Describe the shape of this prism.

d) Sketch a prism with a lesser surface area but the same volume.
Describe the shape of this prism.

Reflect

Suppose you know the area of one face of a rectangular prism.
What else do you need to know to find the volume of the prism? Explain.
Suppose you know the volume of a rectangular prism.
Can you find its dimensions? Use words and diagrams to explain.

Largest Box Problem

Game

YOU WILL NEED

20-cm by 16-cm grids; scissors; tape; rulers

NUMBER OF PLAYERS

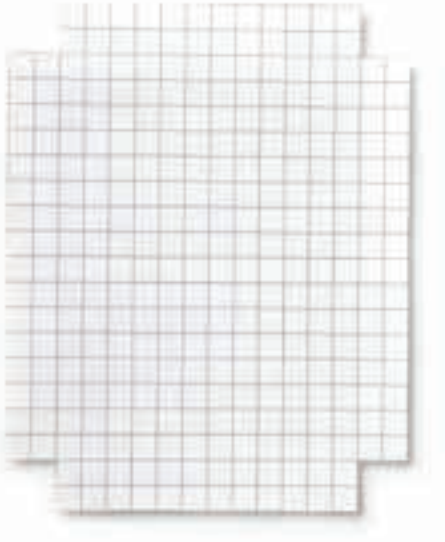
4

GOAL OF THE GAME

To make a box with the greatest volume

HOW TO PLAY

1. Cut congruent squares from the four corners of the grid. Think about what size the squares should be to make a box with the greatest volume.



2. Fold, then tape the sides to form an open box.

3. Measure the length, width, and height of your box. Find its volume.

4. Compare the volume of your box to the volumes of the boxes the other players in your group made. The player whose box has the greatest volume wins.

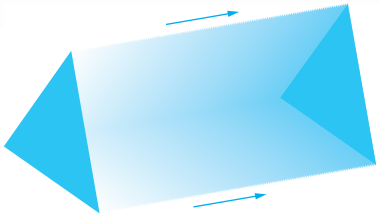
Reflect

- What strategies did you use to make the box with the greatest volume?
- Compare results with another group of students. How do you know you cannot make a box with a greater volume?

4.6 Volume of a Right Triangular Prism

Focus Develop and use a formula to find the volume of a right triangular prism.

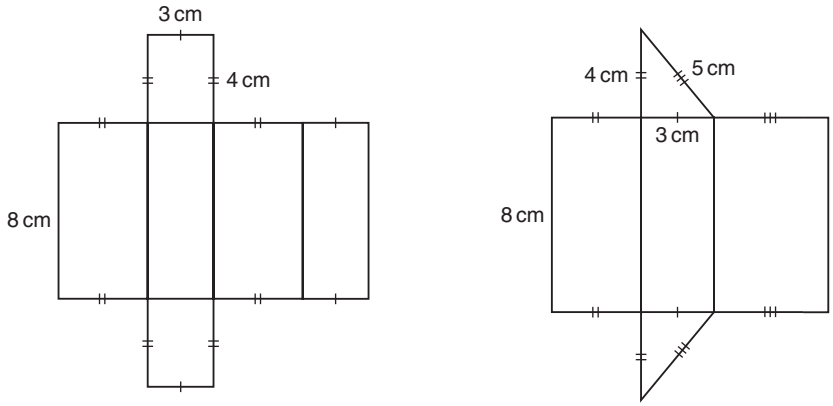
Here is another way to visualize a triangular prism.
A triangle is translated in the air so that each side of the triangle is always parallel to its original position.



How could you use this model to find the volume of the prism?

Investigate

Work with a partner.
You will need scissors and tape.
Your teacher will give you a large copy of these nets.



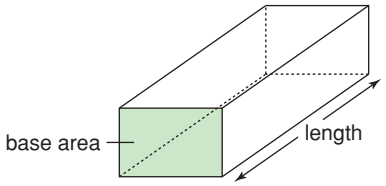
- Identify the prism each net will form.
- Cut out the nets and construct the right prisms.
- Visually compare the volumes of the two prisms.
  How are they related?
- What is the volume of the rectangular prism?
  How can you use this volume to find the volume of the triangular prism?
- What is a formula for the volume of a rectangular prism?
  How can you use this formula to write a formula for the volume of a triangular prism?

Reflect & Share

Combine your prisms with those of another pair of classmates.
How can you arrange the prisms to verify the relationship you found in *Investigate*?

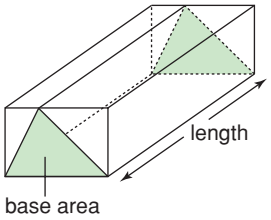
Connect

The volume of a right rectangular prism can be written as:
V = base area $\times$ length



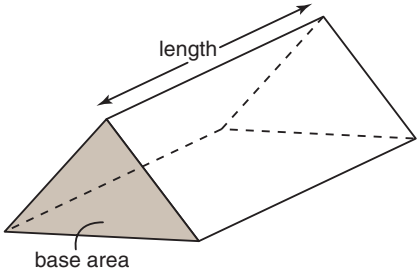
To avoid confusion between the height of a triangle and the height of the prism, use *length* to describe the height of the prism.

Suppose we draw a triangle on the base of the prism so that the base of the triangle is one edge, and the third vertex of the triangle is on the opposite edge.



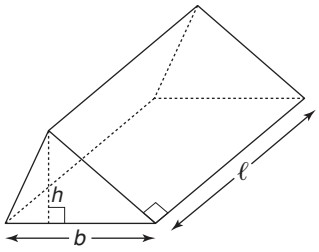
The volume of a triangular prism with this base, and with length equal to the length of the rectangular prism, is one-half the volume of the rectangular prism.

The volume of a right triangular prism
is also: V = base area $\times$ length
The base is a triangle, so the base area is the area of the triangle.



We can use variables to write a formula for the volume of a triangular prism.

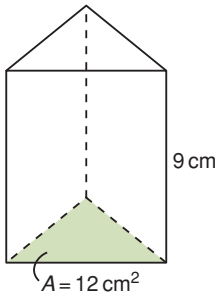
For the triangular prism below:



The length of the prism is ℓ.
Each triangular face has base b and height h.
The volume of the prism is:
V = base area $\times$ length, or $A \times \ell$, or $A\ell$, where $A = \frac{1}{2}\,bh$

Example 1

Find the volume of the prism.



A Solution

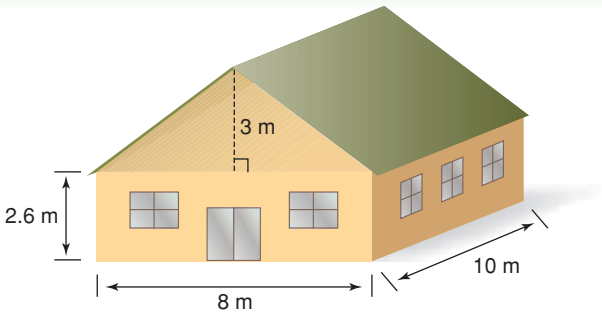
The area of the base of a triangular prism is 12 cm^2.
The length of the prism is 9 cm.
Volume of triangular prism = base area × length

$$V = 12 \times 9$$
$$= 108$$

The volume of the triangular prism is 108 cm^3.

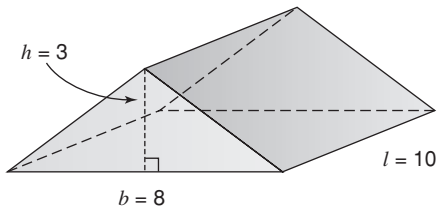
Example 2

Here is a diagram of Renee's new house.
What is the volume of the attic?



A Solution

The attic is a triangular prism.
Sketch the prism.
Use a variable to represent each dimension.
The base of the triangle is: $b = 8$
The height of the triangle is: $h = 3$
The length of the prism is: $\ell = 10$



Use: $V = A\ell$
First find A.
$A = \frac{1}{2}bh$
Substitute: $b = 8$ and $h = 3$

$$A = \frac{1}{2} \times 8 \times 3$$
$$= 12$$

Now find V.
Substitute: $A = 12$ and $\ell = 10$ into $V = A\ell$

$$V = 12 \times 10$$
$$= 120$$

The volume of the attic is 120 m^3.

Discuss the ideas

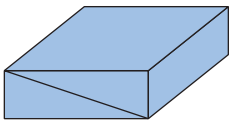
1. A rectangular prism is cut in half to make 2 congruent triangular prisms.
   What do you know about the volume of each triangular prism?
2. Any face can be used as the base of a rectangular prism.
   Can any face be used as the base of a triangular prism? Explain.

Practice

Check

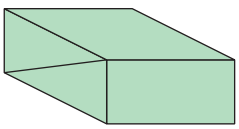
3. Each rectangular prism is divided into 2 congruent triangular prisms along the diagonal shown. The volume of each rectangular prism is given. Find the volume of each triangular prism.

a)



Volume = 450 cm^3

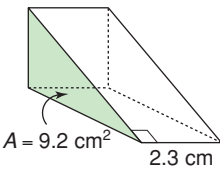
b)



Volume = 624 cm^3

4. The base area and length of each triangular prism are given.
   Find the volume of each prism.

a)



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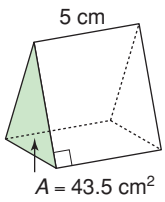

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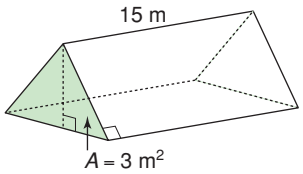
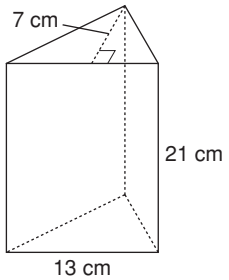


5. Find the volume of each triangular prism.

a)



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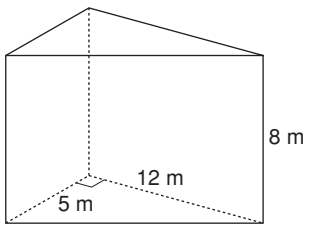

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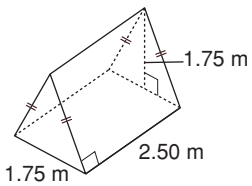
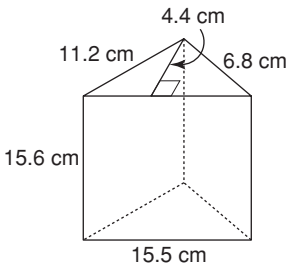


Apply

6. Find the volume of each prism.

a)



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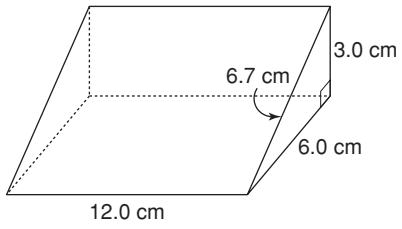
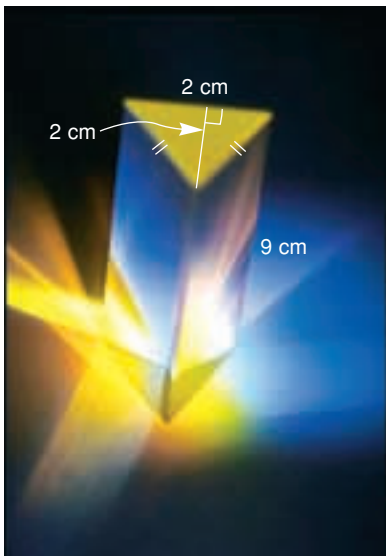


7. What is the volume of glass in this glass prism?



8. The volume of a right triangular prism is 30 cm^3. Each triangular face has area 4 cm^2.
How long is the prism?

9. Assessment Focus

a) Find possible values for A and ℓ for each volume of a right triangular prism. Sketch one possible triangular prism for each volume.

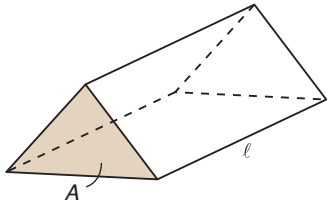
i) 5 cm^3

ii) 9 m^3

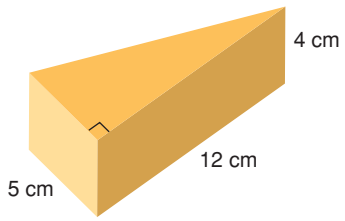
iii) 8 m^3

iv) 18 cm^3

b) How many different prisms can you find in each case?



10. Chico has a wedge of cheddar cheese. He plans to serve the cheese as an appetizer before dinner.

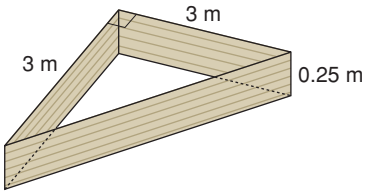


a) What volume of cheese does Chico have?

b) Suppose each person eats 20 cm^3 of cheese.
How many people will the cheese serve?

11. The volume of a triangular prism is 50 m^3. The length of the prism is 5 m. What is the area of each triangular face?

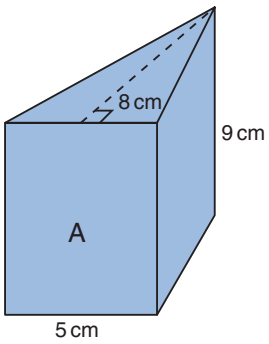
12. Jackie uses this form to build a concrete pad.

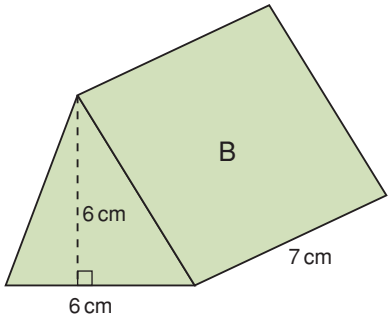


a) How much concrete will Jackie need to mix to fill the form?

b) Suppose Jackie increases the lengths of the equal sides of the form from 3 m to 6 m.
How much more concrete will Jackie need to mix?
Include a diagram.

13. **a)** Predict which triangular prism has the greater volume.

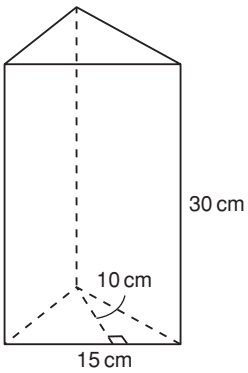




b) Find the volume of each prism.
Was your prediction correct?

c) How could you change one dimension of Prism B so the two prisms have the same volume?

14. **a)** Find the volume of this prism.



b) Suppose the prism contains 1350 mL of water.
What is the depth of the water?

c) What percent of the volume of the prism is water?

15. The volume of a right triangular prism is 198 cm^3. Each triangular face is a right triangle with area 18 cm^2. Find as many dimensions of the prism as you can.

16. Take It Further

A chocolate company produces different sizes of chocolate bars that are packaged in equilateral triangular prisms.
Here is the 100-g chocolate bar.

a) Calculate the surface area and volume of the box.

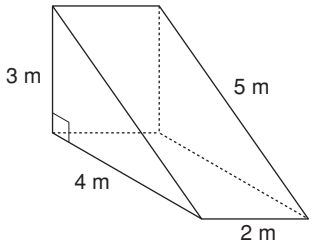
b) The company produces a 400-g chocolate bar.
It has the same shape as the 100-g bar.

i) What are the possible dimensions for the 400-g box? How many different sets of dimensions can you find?

ii) How are the dimensions of the two boxes related, in each case?

17. Take It Further

a) Find the surface area and volume of this triangular prism.



b) What do you think happens to the surface area and volume when the length of the prism is doubled? Justify your prediction.

c) What do you think happens to the surface area and volume when the base and height of the triangular face are doubled? Justify your prediction.

d) What do you think happens to the surface area and volume when all the dimensions are doubled? Justify your prediction.

e) For parts b to d, find the surface area and volume to verify your predictions.

Reflect

How did you use what you know about the volume of a right rectangular prism in this lesson?