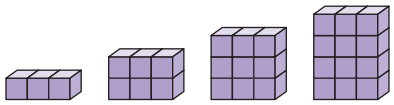
1.4 Relationships in Patterns

Focus Determine a relation to represent a pattern.

Here is a pattern made from linking cubes.



A pattern rule is: Start at 3. Add 3 each time.
This rule relates each term to the term that comes before it.

We can also describe this pattern using the term number.

Term Number	1	2	3	4
Term	3	6	9	12

How does each term relate to the term number?

Explore

On Enviro-Challenge Day, Grade 7 classes compete to see which class can collect the most garbage.

Each student in Ms. Thomson's class pledges to pick up 6 pieces of garbage.

- How many pieces of garbage will be picked up when the number of students is 5? 10? 15? 20? 25? 30?
- What pattern do you see in the numbers of pieces of garbage?
- Write a rule to find how many pieces of garbage will be picked up, when you know the number of students.
- Write an algebraic expression for the number of pieces of garbage picked up by n students.

Reflect & Share

Share your work with another pair of classmates.
Find the number of pieces of garbage picked up by 35 students.
How can you do this using the pattern?
Using the rule? Using the algebraic expression?

Connect

Miss Jackson's class pledges to pick up a total of 10 more pieces of garbage than Ms. Thomson's class.
Here are the numbers of pieces of garbage picked up by different numbers of students.

Number of students	2	4	6	8	10	12
Number of pieces of garbage picked up by Ms. Thomson's class	12	24	36	48	60	72
Number of pieces of garbage picked up by Miss Jackson's class	22	34	46	58	70	82

Pieces of garbage picked up by Miss Jackson's class = 10 + Pieces of garbage picked up by Ms. Thomson's class

Let n represent the number of students who pick up garbage in Ms. Thomson's class.
Then the number of pieces of garbage picked up by Ms. Thomson's class is $6n$.
And, the number of pieces of garbage picked up by Miss Jackson's class is $10 + 6n$.
The number of pieces of garbage is *related* to the number of students.
When we compare or *relate* a variable to an expression that contains the variable, we have a **relation**.
That is, $10 + 6n$ is related to n.

Recall that 10 is the constant term.

This is a relation.

Example

Mr. Prasad plans to hold a party for a group of his friends.
The cost of renting a room is $35.
The cost of food is $4 per person.

a) Write a relation for the cost of the party, in dollars, for n people.

b) How much will a party cost for 10 people? For 15 people?

c) How does the relation change if the cost of food doubles? How much more would a party for 10 people cost? How do you know the answer makes sense?

A Solution

a) The cost of renting a room is \$35.
This does not depend on how many people come.
The cost of food is \$4 per person.
If 5 people come, the cost of food in dollars is: $4 \times 5 = 20$
If n people come, the cost of food in dollars is: $4 \times n$, or $4n$
So, n is related to $35 + 4n$.

b) To find the cost for 10 people, substitute $n = 10$ into $35 + 4n$.

$$\begin{aligned} 35 + 4n &= 35 + 4(10) \\ &= 35 + 40 \\ &= 75 \end{aligned}$$

4(10) means 4 × 10.

The party will cost \$75.

To find the cost for 15 people, substitute $n = 15$ into $35 + 4n$.

$$\begin{aligned} 35 + 4n &= 35 + 4(15) \\ &= 35 + 60 \\ &= 95 \end{aligned}$$

The party will cost \$95.

c) If the cost of food doubles, Mr. Prasad will pay \$8 per person.
If n people come, the cost for food, in dollars, is $8n$.
For n people, the cost of the party, in dollars, is now $35 + 8n$.
If 10 people come, the cost is now:

$$\begin{aligned} 35 + 8n &= 35 + 8(10) \\ &= 35 + 80 \\ &= 115 \end{aligned}$$

The party will cost \$115.
This is an increase of $\$115 - \$75 = \$40$.
The answer makes sense because the cost is now \$4 more per person.
So, the extra cost for 10 people would be $\$4 \times 10$, or \$40 more.

Math Link

History
The word "algebra" comes from the Arabic word "al-jabr." This word appeared in the title of one of the earliest algebra texts, written around the year 825 by al-Khwarizmi. He lived in what is now Uzbekistan.

Practice

1. **i)** For each number pattern, how is each term related to the term number?
ii) Let n represent any term number. Write a relation for the term.

a)

Term Number	1	2	3	4	5	6
Term	2	4	6	8	10	12

b)

Term Number	1	2	3	4	5	6
Term	3	4	5	6	7	8

c)

Term Number	1	2	3	4	5	6
Term	8	16	24	32	40	48

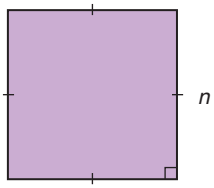
d)

Term Number	1	2	3	4	5	6
Term	6	7	8	9	10	11

2. There are n students in a class. Write a relation for each statement.
a) the total number of pencils, if each student has three pencils
b) the total number of desks, if there are two more desks than students
c) the total number of geoboards, if each pair of students shares one geoboard
d) the total number of stickers, if each student gets four stickers and there are ten stickers left over

3. A person earns $10 for each hour worked.
a) Write a relation for her earnings for n hours of work.
b) How much does she earn for 30 h of work?

4. **a)** Write a relation for the perimeter of a square with side length n centimetres.
b) What is the perimeter of a square with side length 12 cm?
c) Suggest a situation that could be represented by each relation.
i) $3s$ is related to s **ii)** $8t$ is related to t



5. Suggest a real-life situation that could be represented by each relation.
a) $n + 5$ is related to n **b)** $15 + 2p$ is related to p
c) $3t + 1$ is related to t
How do you know each situation fits the relation?

6. Koko is organizing an overnight camping trip. The cost to rent a campsite is $20. The cost of food is $9 per person.

a) How much will the trip cost if 5 people go? 10 people go?

b) Write a relation for the cost of the trip when p people go.

c) Suppose the cost of food doubles.
Write a relation for the total cost of the trip for p people.

d) Suppose the cost of the campsite doubles.
Write a relation for the total cost of the trip for p people.

e) Explain why using the variable p is helpful.

7. **Assessment Focus** A pizza with cheese and tomato toppings costs $8.00. It costs $1 for each extra topping.

a) Write a relation for the cost of a pizza with e extra toppings.

b) What is the cost of a pizza with 5 extra toppings?

c) On Tuesdays, the cost of the same pizza with cheese and tomato toppings is $5.00. Write a relation for the cost of a pizza with e extra toppings on Tuesdays.

d) What is the cost of a pizza with 5 extra toppings on Tuesdays?

e) How much is saved by buying the pizza on Tuesday?

8. Write a relation for the pattern rule for each number pattern. Let n represent any term number.

a) 4, 8, 12, 16, … **b)** 7, 8, 9, 10, … **c)** 0, 1, 2, 3, …

9. **Take It Further**

i) For each number pattern, how is each term related to the term number?

ii) Let n represent any term number. Write a relation for the term.

a)

Term Number	1	2	3	4	5	6
Term	3	5	7	9	11	13

b)

Term Number	3	4	5	6	7	8
Term	7	10	13	16	19	22

c)

Term Number	2	3	4	5	6	7
Term	5	9	13	17	21	25

Reflect

How did your knowledge of patterning help you in this lesson?