6.1 Solving Equations Using Models

Focus Use concrete materials to model and solve linear equations.

Recall that one yellow unit tile represents +1.
One red unit tile represents −1.
What happens when you combine one red unit tile and one yellow unit tile?

The yellow variable tile represents any variable, such as n or x.

Investigate

Work with a partner.

Marie received three $100 savings bonds on her first birthday.
Her grandmother promised to give her 2 savings bonds each year after that for her birthday. How old will Marie be when she has 13 savings bonds?

Let n represent Marie's age in years.
Write an equation you can use to solve for n.
Use tiles to represent the equation.
Use the tiles to solve the equation.
Sketch the tiles you used.

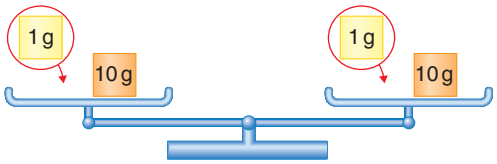
Reflect & Share

Compare your equation and its solution with those of another pair of classmates.
Did you write the same equation?
If your answer is yes, find another way to write the equation.
If your answer is no, are both equations correct?
How do you know?
Share your strategies for solving the equation using tiles.

Connect

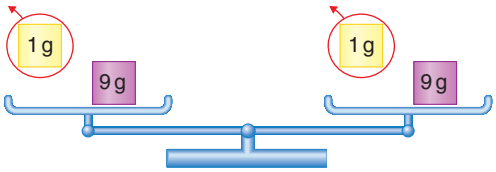
We can use a balance-scales model to solve an equation.

To keep the scales balanced, we must do the same to both sides.
For example,
we add the same mass:



The scales remain balanced.

or remove the same mass:



The scales remain balanced.

Example 1

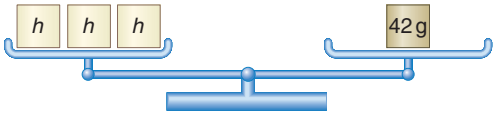
Herman is in the last round of the spell-a-thon in his school.
A contestant receives 3 points for every word spelled correctly.
Herman has 42 points. How many words has he spelled correctly?

A Solution

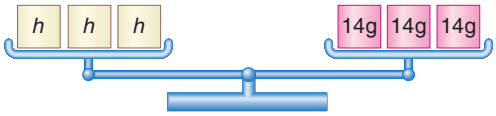
Let h represent the number of words Herman has spelled correctly.
Then, the number of points is 3 times h, or $3h$.

Since Herman has 42 points, the algebraic equation is $3h = 42$.
Use a balance-scales model to represent this equation.

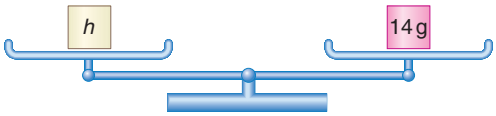
On the left side, show masses to represent $3h$.
On the right side, show a mass to represent 42.



Since there are 3 identical unknown masses in the left pan, replace 42 g in the right pan with 3 equal masses.
Each mass is 14 g.



So, each unknown mass is 14 g.
$h = 14$



Herman has spelled 14 words correctly.

Check: 14 words worth 3 points each is $14 \times 3 = 42$ points.
The solution is correct.

Another strategy to solve an equation is to use algebra tiles. We rearrange the tiles to end up with variable tiles on one side and unit tiles on the other side.

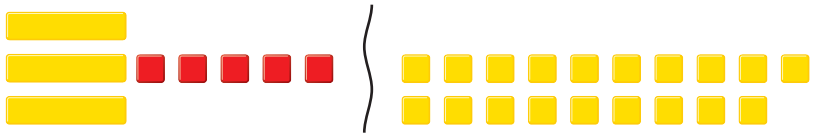
Example 2

Jodee is also a contestant in the spell-a-thon.
A contestant receives 3 points for every word spelled correctly.
Because of a technical penalty, Jodee loses 5 points. She now has 19 points.
How many words has Jodee spelled correctly?

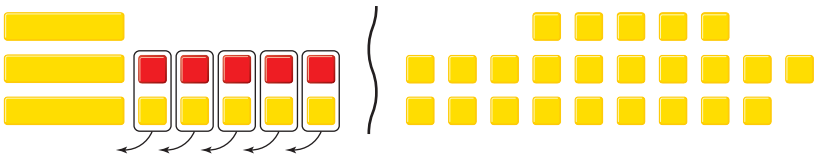
A Solution

Let j represent the number of words Jodee has spelled correctly.
Then, the number of points she receives is $3j$.
When the penalty is considered, the number of points is $3j - 5$.
So, the equation is: $3j - 5 = 19$

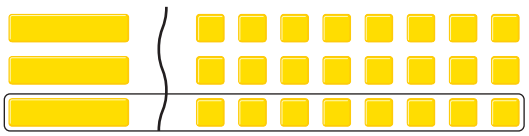
On the left side, place tiles to represent $3j - 5$.
On the right side, place tiles to represent 19.



To isolate the j-tiles on the left side, add 5 positive unit tiles to make zero pairs.
To keep the balance, add 5 positive unit tiles to the right side, too.

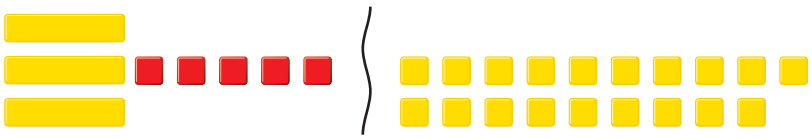


There are 3 j-tiles. So, arrange the unit tiles into 3 equal groups.

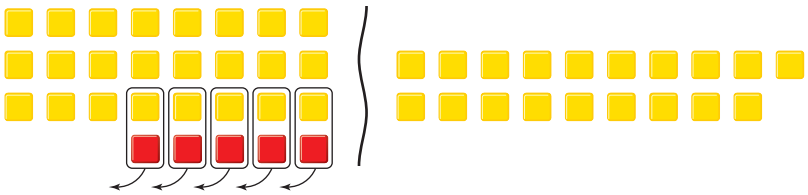


The solution is $j = 8$.
Jodee has spelled 8 words correctly.

We can verify the solution by replacing each positive variable tile with 8 positive unit tiles. Then:



becomes:



Since there are now 19 positive unit tiles on each side, the solution is correct.

Many different types of equations can be modelled and solved with algebra tiles.

The opposite of x is $-x$.
So, the red variable tile  represents $-x$.
As with the unit tiles, a positive variable tile and a negative variable tile combine to model 0.

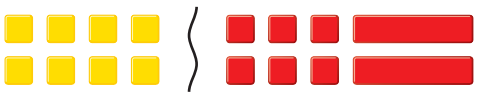
To make some equations easier to solve, it helps to make the variables positive.

Example 3

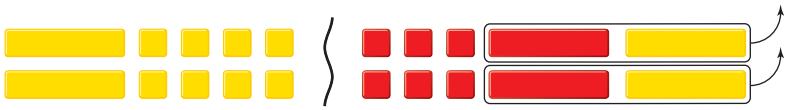
Use algebra tiles to solve: $8 = -6 - 2x$
Verify the solution.

A Solution

Place tiles to model the equation.

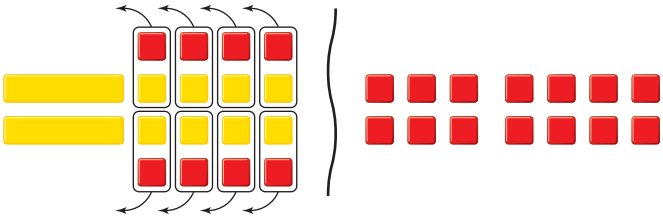


To make the variable tiles positive, add 2 positive variable tiles to each side.

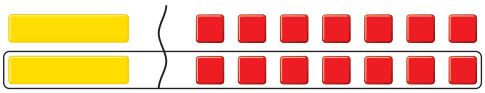


These tiles remain.

To isolate the variable tiles, add 8 negative unit tiles to each side.
Remove zero pairs.



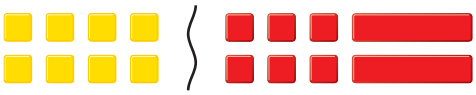
Arrange the tiles remaining on each side into 2 equal groups.



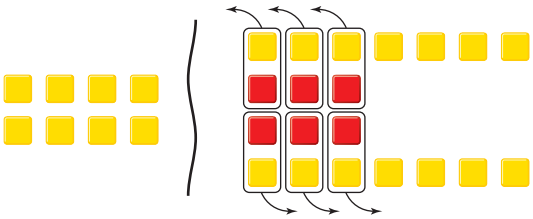
The solution is $x = -7$.

We can verify the solution.
The original equation has negative variable tiles.
If $x = -7$, then $-x = -(-7) = 7$
So, replace each variable tile in the original equation with 7 positive unit tiles.

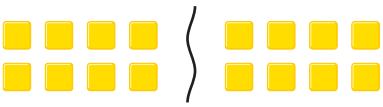
Then:



becomes:



Remove zero pairs.



Since there are now 8 positive unit tiles on each side, the solution is correct.

Discuss the ideas

1. In *Example 1*, we used h to represent the number of words Herman spelled correctly. Why do you think we used h? Could we have used a different letter? Explain.
2. In *Example 2*, we solved the equation with tiles to represent $3j - 5$ on the left side and 19 on the right side. Could we have solved the equation with 19 on the left side and $3j - 5$ on the right side? Would the solution have been different? Justify your answer.
3. In *Example 3*, how would you solve the equation by leaving the variable tiles on the right side of the equation?
4. Could we have solved the equation in *Example 3* using a balance-scales model? Why or why not?

Practice

Check

Use a model to solve each equation.

5. Draw pictures to represent the steps you took to solve each equation.
   a) $4s = 16$ b) $5t = -15$
   c) $18 = 6a$ d) $-18 = 3b$

6. Draw pictures to represent the steps you took to solve each equation.
   a) $3x + 2 = 8$ b) $4s - 3 = 9$
   c) $10 = 6c + 4$ d) $-4 = 5m + 6$

7. Three more than six times a number is 21. Let n represent the number.
   a) Write an equation you can use to solve for n.
   b) Represent the equation with tiles. Use the tiles to solve the equation. Sketch the tiles you used.
   c) Verify the solution.

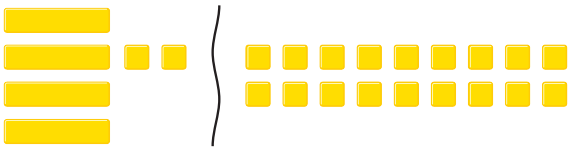
Apply

8. Three less than six times a number is 21. Let n represent the number.
   a) Write an equation you can use to solve for n.
   b) Represent the equation with tiles. Use the tiles to solve the equation. Sketch the tiles you used.
   c) Verify the solution.

9. Maeve wants her friend to guess how many cards she has in her hand. She says that if the number of cards in her hand is tripled, and 4 are added, then she has 22 cards.
   a) Choose a variable. Write an equation for this situation.
   b) Use a balance-scales model to solve the equation.
   c) Verify the solution. Show how you did this.

10. Curtis is practising modelling equations. He is trying to model the equation: $4x - 2 = 18$
Curtis begins by using algebra tiles.

a) Check Curtis' work. Is this the correct model? Explain.

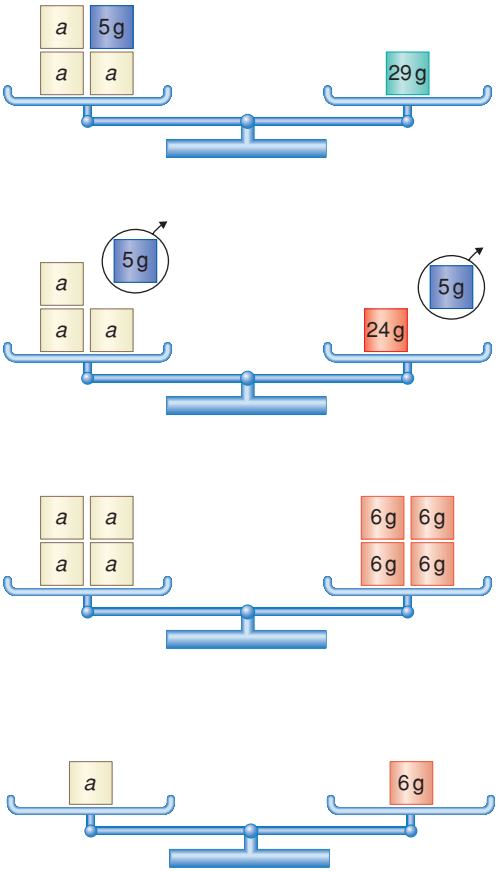


b) If your answer to part a is yes, use the tiles to try to solve the equation. If your answer to part a is no, describe the error, correct it, then use algebra tiles to solve the equation.

11. Use a model to solve each equation. Verify the solution.

a) $-2x = -6$
b) $-15 = 3x$
c) $-24 = -4x$
d) $9x = -27$

12. **Assessment Focus** Breanna and 3 friends need \$29 to buy a game. Breanna has \$5. Each friend will contribute an equal amount of money. Breanna wants to know how much money each friend should contribute. She uses a to represent this amount in dollars.
Breanna is trying to model and solve the equation: $3a + 5 = 29$



Breanna's solution is $a = 6$.
But, when she checks it, she notices that 3 times \$6 plus her \$5 is \$18 + \$5, which is \$23.
The money is not enough to buy the game.

a) Find Breanna's mistake.
b) Sketch the balance scales to model the correct solution.
c) Verify the solution.

13. Use a model to solve each equation. Verify the solution.

a) $-2x + 3 = 13$

b) $-2x - 3 = -13$

c) $2x - 3 = -13$

d) $2x + 3 = -13$

14. Roger brings 4 cakes for dessert to the community potluck feast and powwow. His son brings 2 individual servings of dessert. Altogether, there will be 34 people at the feast. Each person has 1 serving of dessert.

a) Choose a variable to represent the number of pieces into which each cake must be cut.
Write an equation to describe this situation.

b) Use a model to solve the equation.

c) Verify the solution.
Show your work.

15. **Take It Further** A pattern rule for a number pattern is represented by $5 - 8n$, where n is the term number. What is the term number for each term value?

a) -3

b) -35

c) -155

16. **Take It Further**

a) Write an equation you could solve with balance scales and with algebra tiles.

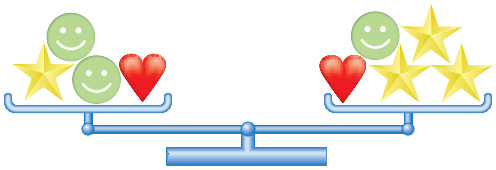
b) Write an equation you could solve with algebra tiles but not with balance scales.

c) Is there an equation that could be solved with balance scales but not with algebra tiles?

Justify your answer each time.

17. **Take It Further**

a) Write an equation in words to describe these balance scales.



b) **i)** The mass of a star is 11 g. What is the mass of a smiley face?
What is the mass of a heart?
Justify your answers.

ii) Compare the strategy you used in part a with that of another classmate. If the strategies are different, is one more efficient than the other?

c) Verify your answers. Write to explain your thinking.

Reflect

You have learned two models to solve an equation.
Are there situations where you prefer one model over the other?
Give an example.