3.1 Using Models to Multiply Fractions and Whole Numbers

Focus Use models to multiply a fraction by a whole number.

How many ways can you find this sum?
$2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2$

Investigate

Work with a partner.
Use any models to help.

Each of 4 students needs $\frac{5}{6}$ of a bag of oranges to make a pitcher of freshly squeezed orange juice.
Each bag of oranges contains 12 oranges.
How many bags of oranges are used?

Reflect & Share

Compare your strategy for solving the problem with that of another pair of classmates.
Did you use addition to solve the problem?
If so, which addition expression did you use?
How could you represent the problem with a multiplication expression?

Connect

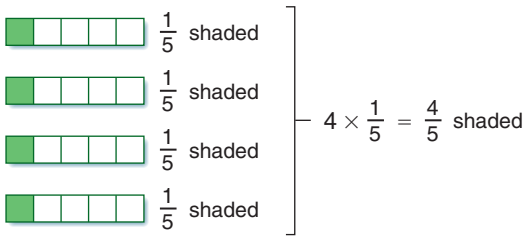
We can find a meaning for: $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{4}{5}$

➤ Repeated addition can be written as multiplication.
$\frac{1}{5}$ is added 4 times.

$\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = 4 \times \frac{1}{5} = \frac{4}{5}$

$\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{1}{5} \times 4 = \frac{4}{5}$

We can show this as a picture.



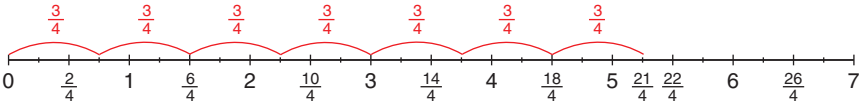
Similarly: $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{21}{4}$

But, $\frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = 7 \times \frac{3}{4} = \frac{21}{4}$

So, $7 \times \frac{3}{4} = \frac{21}{4}$

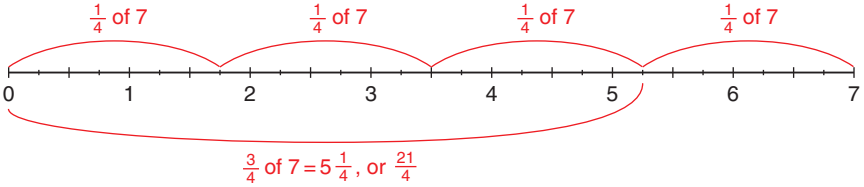
Also, $\frac{3}{4} \times 7 = \frac{21}{4}$

➤ We can use a number line divided into fourths to show that $7 \times \frac{3}{4} = \frac{21}{4}$.

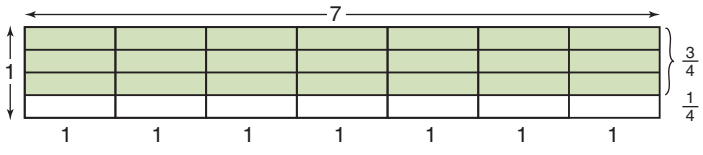


We can use a different number line to show that $\frac{3}{4} \times 7 = \frac{21}{4}$.

For $\frac{3}{4} \times 7$, we can think: $\frac{3}{4}$ of 7



➤ Another way to multiply $7 \times \frac{3}{4}$:
Sketch a rectangle with base 7 units and height 1 unit.
Divide the height into fourths.
Shade the rectangle with base 7 and height $\frac{3}{4}$.



The area of the shaded rectangle is: base × height $= 7 \times \frac{3}{4}$

Each small rectangle has area: $1 \times \frac{1}{4} = \frac{1}{4}$

So, the area of the shaded rectangle is: $21 \times \frac{1}{4} = \frac{21}{4}$

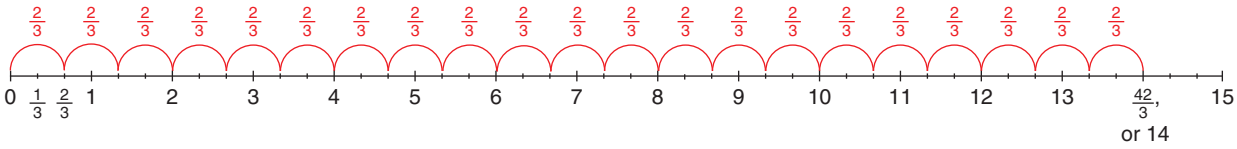
Then, $7 \times \frac{3}{4} = \frac{21}{4}$ **This is a multiplication equation.**

Example 1

New flooring has been installed in two-thirds of the classrooms in the school.
There are 21 classrooms in the school.
How many classrooms have new flooring?

A Solution

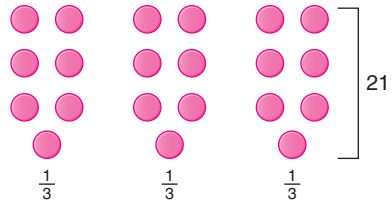
Multiply: $21 \times \frac{2}{3}$
Use a number line divided into thirds.



So, $21 \times \frac{2}{3} = \frac{42}{3}$, or 14
Fourteen classrooms have new flooring.

Example 1
Another Solution

Find: $\frac{2}{3}$ of 21
Use counters.
Model 21 with counters.



Find thirds by dividing the counters into 3 equal groups.
Each group contains 7 counters.
$\frac{1}{3}$ of $21 = 7$
So, $\frac{2}{3}$ of $21 = 14$
Fourteen classrooms have new flooring.

Example 2

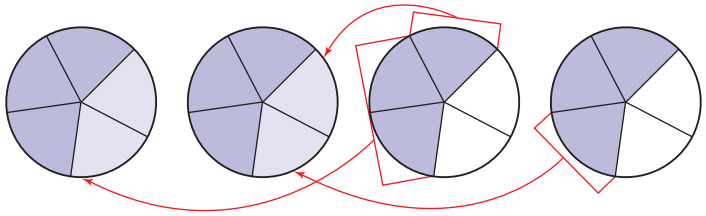
An office building with four floors has rented out $\frac{3}{5}$ of each floor.
How many floors of the building have been rented?

A Solution

Multiply: $4 \times \frac{3}{5}$

$4 \times \frac{3}{5} = \frac{3}{5} + \frac{3}{5} + \frac{3}{5} + \frac{3}{5}$

Model the expression $\frac{3}{5} + \frac{3}{5} + \frac{3}{5} + \frac{3}{5}$ with fraction circles.



Put the fifths together to make wholes.
2 wholes and two fifths equal $2\frac{2}{5}$.

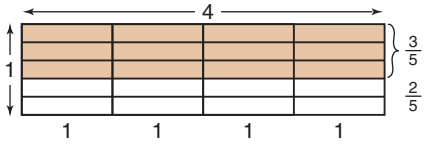
So, $4 \times \frac{3}{5} = 2\frac{2}{5}$

$2\frac{2}{5}$ floors of the office building have been rented.

Example 2
Another Solution

Multiply: $4 \times \frac{3}{5}$

Sketch a rectangle with base 4 units and height 1 unit.
Divide the height into fifths.
Shade the rectangle with base 4 and height $\frac{3}{5}$.



The area of the shaded rectangle is:
base $\times$ height $= 4 \times \frac{3}{5}$

Each small rectangle has area: $1 \times \frac{1}{5} = \frac{1}{5}$

So, the shaded area is: $12 \times \frac{1}{5} = \frac{12}{5}$, or $2\frac{2}{5}$

So, $4 \times \frac{3}{5} = 2\frac{2}{5}$

$2\frac{2}{5}$ floors of the office building have been rented.

Discuss the ideas

1. Why can a product be written as repeated addition?
2. When might you not want to use repeated addition to find a product?
3. How could you use a rectangle model to solve the problem in *Example 1*?
4. How could you use a number line to solve the problem in *Example 2*?

Practice

Check

5. Write each statement as a multiplication statement in two ways.

a) $\frac{5}{9}$ of 45 **b)** $\frac{3}{8}$ of 32

c) $\frac{1}{12}$ of 36 **d)** $\frac{4}{5}$ of 25

6. Write each repeated addition as a multiplication statement in two ways.

a) $\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

b) $\frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5}$

c) $\frac{3}{10} + \frac{3}{10} + \frac{3}{10} + \frac{3}{10}$

7. Use fraction circles to find: $\frac{2}{3} \times 6$

a) Write the multiplication as repeated addition.

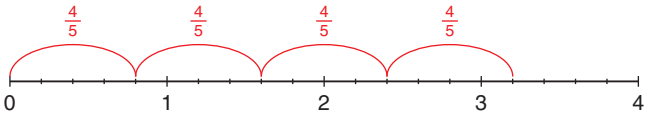
b) Use fraction circles to find the sum.

c) Sketch the fraction circles.

d) Write the multiplication equation the fraction circles represent.

8. Write the two multiplication equations each number line represents.

a)



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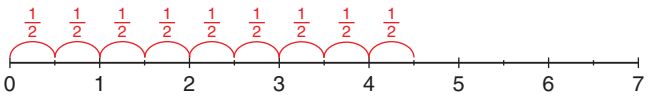
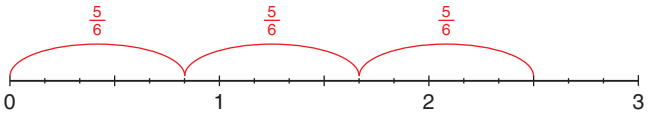


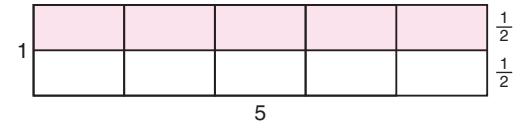
c)



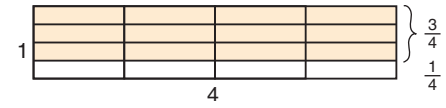
Apply

9. For each diagram below, state the product the shaded area represents.

a)

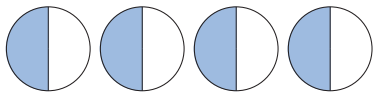


b)



10. Write the two multiplication statements each set of fraction circles represents. Then find each product.

a)



b)

11. Use fraction circles to find each product. Sketch the fraction circles. Write a multiplication equation each time.

a) $5 \times \frac{1}{8}$ **b)** $\frac{2}{5} \times 3$ **c)** $4 \times \frac{5}{12}$

12. Use counters to help you find each product.

a) $\frac{1}{2} \times 24$ **b)** $\frac{1}{3} \times 24$ **c)** $\frac{1}{4} \times 24$

d) $\frac{1}{6} \times 24$ **e)** $\frac{1}{8} \times 24$ **f)** $\frac{1}{12} \times 24$

13. Use the results in question 12 to find each product.

a) $\frac{2}{2} \times 24$ **b)** $\frac{2}{3} \times 24$ **c)** $\frac{3}{4} \times 24$

d) $\frac{5}{6} \times 24$ **e)** $\frac{3}{8} \times 24$ **f)** $\frac{5}{12} \times 24$

14. Multiply. Draw a picture or number line to show each product.

a) $3 \times \frac{4}{7}$ **b)** $\frac{2}{15} \times 10$

c) $4 \times \frac{9}{4}$ **d)** $\frac{2}{5} \times 7$

15. Draw and shade rectangles to find each product.

a) $\frac{1}{3} \times 12$ **b)** $\frac{1}{5} \times 15$

c) $\frac{3}{5} \times 15$ **d)** $\frac{3}{8} \times 16$

16. Multiply.

a) $3 \times \frac{4}{5}$ **b)** $5 \times \frac{7}{9}$ **c)** $\frac{5}{3} \times 6$

d) $\frac{1}{2} \times 5$ **e)** $12 \times \frac{7}{8}$ **f)** $\frac{2}{4} \times 9$

17. It takes $\frac{2}{3}$ h to pick all the apples on one tree at Springwater Farms. There are 24 trees. How long will it take to pick all the apples? Show your work.

18. **Assessment Focus**

a) Describe a situation that could be represented by $5 \times \frac{3}{8}$.

b) Draw a picture to show $5 \times \frac{3}{8}$.

c) What meaning can you give to $\frac{3}{8} \times 5$?
Draw a picture to show your thinking.

19. Parri used the expression $\frac{5}{8} \times 16$ to solve a word problem.
What might the word problem be?
Solve the problem.

20. Naruko went to the West Edmonton Mall. She took $28 with her. She spent $\frac{4}{7}$ of her money on rides. How much money did Naruko spend on rides? Use a model to show your answer.

21. **Take It Further**

a) Use models. Multiply.

i) $2 \times \frac{1}{2}$ **ii)** $3 \times \frac{1}{3}$

iii) $4 \times \frac{1}{4}$ **iv)** $5 \times \frac{1}{5}$

b) Look at your answers to part a.
What do you notice?
How can you explain your findings?

c) Write two different multiplication statements with the same product as in part a.

22. **Take It Further** Jacques takes $\frac{3}{4}$ h to fill one shelf at the supermarket.
Henri can fill the shelves in two-thirds Jacques' time.
There are 15 shelves. Henri and Jacques work together.
How long will it take to fill the shelves?
Justify your answer.

Reflect

Explain how your knowledge of adding fractions helped you in this lesson.
Include an example.