

# 5.3

## Solving Percent Problems

**Focus** Find the whole, when given a percent, and find the percent increase and decrease.

### Investigate

Work with a partner.

Tasha conducted a survey of the students in her school.

- From the results, Tasha calculated that 60% of the students go to school by bus.
- Liam knows that 450 students go to school by bus. How can Liam use these data to find the number of students in the school?
- Tasha also found that 50% more students go by bus than walk or drive.

About how many students walk or drive to school?

Sketch number lines to illustrate your work.



### Reflect & Share

Compare your results with those of another pair of classmates. Discuss the strategies you used to solve the problems.



## Connect

Grady is 13 years old and 155 cm tall.

His height at this age is about 90% of his final height.

To estimate Grady's final height:

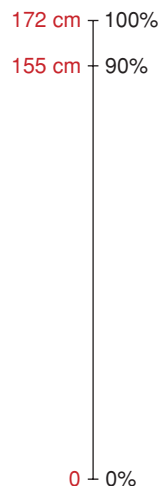
90% of Grady's height is 155 cm.

So, 1% of his height is:  $\frac{155 \text{ cm}}{90}$

And, 100% of his height is:  $\frac{155 \text{ cm}}{90} \times 100 \doteq 172.2 \text{ cm}$

So, Grady's final height will be about 172 cm.

When we know a percent of the whole, we divide to find 1%, then multiply by 100 to find 100%, which is the whole.



### Example 1

Find the number in each case.

- a) 70% of a number is 63.
- b) 175% of a number is 105.

#### A Solution

- a) 70% of a number is 63.

So, 1% of the number is:

$$\frac{63}{70} = 0.9$$

And, 100% of the number is:

$$0.9 \times 100 = 90$$

The number is 90.

- b) 175% of a number is 105.

So, 1% of the number is:

$$\frac{105}{175} = 0.6$$

And, 100% of the number is:

$$0.6 \times 100 = 60$$

The number is 60.

## Example 2

- a) A length of 30 cm increased by 40%. What is the new length?
- b) A mass of 50 g decreased by 17%. What is the new mass?

### A Solution

- a) The length increased by 40%.

So, the increase in length is 40% of 30.

First, write 40% as a decimal.

$$\begin{aligned}40\% &= \frac{40}{100} \\ &= 0.4\end{aligned}$$

$$\begin{aligned}\text{Then, } 40\% \text{ of } 30 &= 0.4 \times 30 \\ &= 12\end{aligned}$$

The length increased by 12 cm.

So, the new length is:  $30 \text{ cm} + 12 \text{ cm} = 42 \text{ cm}$

- b) The mass decreased by 17%.

So, the decrease in mass is 17% of 50.

First, write 17% as a decimal.

$$17\% = \frac{17}{100}, \text{ or } 0.17$$

$$\begin{aligned}\text{Then, } 17\% \text{ of } 50 &= 0.17 \times 50 \\ &= 8.5\end{aligned}$$

The mass decreased by 8.5 g.

So, the new mass is:  $50 \text{ g} - 8.5 \text{ g} = 41.5 \text{ g}$

### Example 2

#### Another Solution

- a) The length increased by 40%.

So, the new length is  $100\% + 40\% = 140\%$  of the original length.

Find 140% of 30.

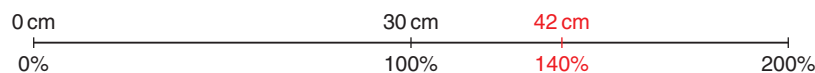
First, write 140% as a decimal.

$$140\% = \frac{140}{100}, \text{ or } 1.4$$

$$\begin{aligned}\text{Then, } 140\% \text{ of } 30 &= 1.4 \times 30 \\ &= 42\end{aligned}$$

The new length is 42 cm.

We can illustrate this answer on a number line.



- b) The mass decreased by 17%.

So, the new mass is  $100\% - 17\% = 83\%$  of the original mass.

Find 83% of 50.

First, write 83% as a decimal.

$$83\% = \frac{83}{100}$$

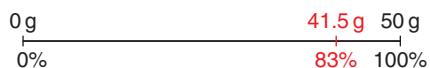
$$= 0.83$$

Then, 83% of 50 =  $0.83 \times 50$

$$= 41.5$$

The new mass is 41.5 g.

We can illustrate this answer on a number line.



Another type of problem involving percents is to find the **percent increase** or **percent decrease**.

### Example 3

- a) The price of a carton of milk in the school cafeteria increased from 95¢ to \$1.25.  
What was the percent increase in price?
- b) The price of a green salad decreased from \$2.50 to \$1.95.  
What was the percent decrease in price?



### A Solution

Write \$1.25 in cents:  $\$1.25 = 125\text{¢}$

- a) The increase in price was:  $125\text{¢} - 95\text{¢} = 30\text{¢}$

To find the percent increase, write the increase as a fraction of the original price:  $\frac{30\text{¢}}{95\text{¢}}$

To write this fraction as a percent:  $\frac{30}{95} \doteq 0.32$

Use a calculator.

$$= \frac{32}{100}$$

$$= 32\%$$

The price of a carton of milk increased by about 32%.



b) The decrease in price is:  $\$2.50 - \$1.95 = \$0.55$ , or 55¢

To find the percent decrease, write the decrease as

a fraction of the original price:  $\frac{55¢}{\$2.50} = \frac{55¢}{250¢}$

$$\$2.50 = 250¢$$

To write this fraction as a percent:  $\frac{55}{250} = 0.22$

Use a calculator.

$$= \frac{22}{100}$$

$$= 22\%$$

The price of a green salad decreased by 22%.



## Discuss the ideas

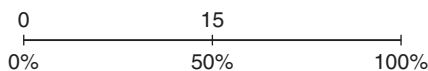
1. In *Example 2*, you learned two methods to find the new length and mass. Which method do you think you would prefer to use in the *Practice* questions that follow? Explain.
2. In *Example 3*, how could you check that the percent increase and percent decrease are correct?

## Practice

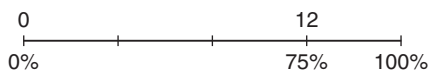
### Check

3. Use the number line to find each number.

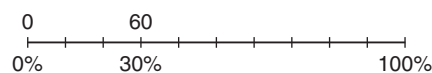
a) 50% of a number is 15.



b) 75% of a number is 12.



c) 30% of a number is 60.



d) 80% of a number is 120.



4. Find the number in each case.

Illustrate each answer with a number line.

a) 25% of a number is 5.

b) 75% of a number is 18.

c) 4% of a number is 32.

d) 120% of a number is 48.

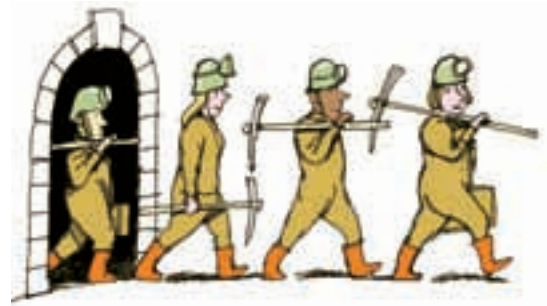
- 5.** Write each increase as a percent. Illustrate each answer with a number line.
- The elastic band stretched from 5 cm to 10 cm.
  - The price of a haircut increased from \$8.00 to \$12.00.
- 6.** Write each decrease as a percent. Illustrate each answer with a number line.
- The price of a book decreased from \$15.00 to \$12.00.
  - The number of students who take the bus to school decreased from 200 to 150.

### Apply

- 7.** Find the whole amount in each case.
- 15% is 125 g.
  - 9% is 45 cm.
  - 0.8% is 12 g.
- 8.** Write each increase as a percent. Illustrate each answer with a number line.
- The price of a house increased from \$320 000 to \$344 000.
  - The area of forest in southwestern Yukon affected by the spruce bark beetle increased from 41 715 ha in 2003 to 99 284 ha in 2004.

One hectare (1 ha) is a unit of area equal to 10 000 m<sup>2</sup>.

- 9.** Write each decrease as a percent. Illustrate each answer with a number line.
- The price of gasoline decreased from 109.9¢/L to 104.9¢/L.
  - The number of students in the class who listen to MP3 players decreased from 17 to 10.
- 10.** There were about 193 000 miners in Canada in 1986. By 2001, the number of miners was 12% less. How many miners were there in 2001?



- 11.** The world's tallest totem pole, known as the Spirit of Lekwammen, was raised on 04 August, 1994 at Victoria, BC, prior to the Commonwealth Games. The totem pole stood about 55 m tall. A local airport was concerned that seaplanes might hit it. So, in 1998 it was partially dismantled. It then stood about 12 m tall. Find the percent decrease in the height of the totem pole.



**12.** Olivia has 2 puppies, George and Jemma. Each puppy had a birth mass of 1.5 kg. At the end of Week 1, Jemma's mass was 15% greater than her birth mass. At the end of Week 2, Jemma's mass was 15% greater than her mass after Week 1. At the end of Week 2, George's mass was 30% greater than his birth mass.

- What was each puppy's mass after Week 2?
- Why are the masses in part a different?

**13. Assessment Focus** In 1990, the population of Calgary, Alberta, was about 693 000. The population increased by about 24% from 1990 to 2000. From 2000 to 2005, the population increased by about 11%.

- In 2000, about how many people lived in Calgary?
- In 2005, about how many people lived in Calgary?
- Write the increase in population from 1990 to 2005 as a percent of the 1990 population.
- Is your answer in part c 35%? Should the answer be 35%? Explain why or why not.



**14.** In 2004, the crime rate in a city was 15 194 crimes per 100 000 population. The crime rate decreased by 6% in 2005, and by 4% in 2006.

- What was the crime rate at the end of 2006?
- Is your answer to part a the same as a decrease in the crime rate of 10%? Why or why not?

**15.** On average, a girl reaches 90% of her final height when she is 11 years old, and 98% of her final height when she is 17 years old.

- Anna is 11 years old. She is 150 cm tall. Estimate her height when she is 20 years old.
- Raji is 17 years old. She is 176 cm tall. Estimate her height when she is 30 years old. What assumptions do you make?

**16.** On average, a boy reaches 90% of his final height when he is 13 years old, and 98% of his final height when he is 18 years old. Use these data or the data in question 15 to estimate your height when you are 21 years old.

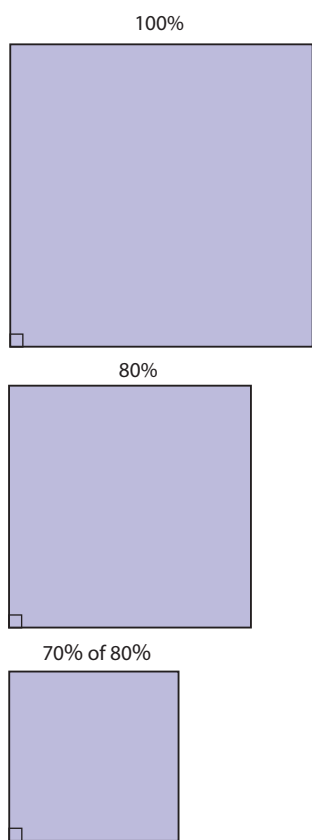
Explain any assumptions you make. Show your work.

**17.** After a price reduction of 20%, the sale price of an item is \$16. A student says, "So, the original price must have been 120% of the sale price."

Is this statement correct?

Justify your answer.

- 18. Take It Further** A photocopier is used to reduce a square. When the photocopier is set at 80%, the side length of the copy is 80% of its length on the original square. Suppose the side length of the square is 10 cm. It is copied at 80%. The image square is then copied again at 70%.



- a) What is the side length on the final image square?  
 b) What is the percent decrease in the side length of the square?  
 c) What is the area of the final image square?  
 d) What is the percent decrease in the area of the square?

- 19. Take It Further** A box was  $\frac{3}{4}$  full of marbles. The box fell on the floor. Thirty marbles fell out. This was 20% of the marbles in the box. How many marbles would a full box contain?
- 20. Take It Further** Shen dug a 5-m by 15-m garden along one side of his rectangular lawn. He says that this has reduced the area of his lawn by 25%. What are the dimensions of the remaining lawn? Use a diagram to show your answer. Describe the strategy you used to solve the problem. What assumptions do you make?

## Reflect

What is the difference between a percent increase and a percent decrease? Include examples in your explanation.