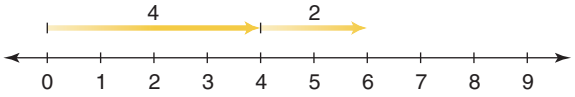
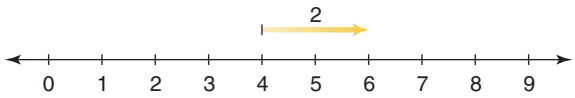
2.3 Adding Integers on a Number Line

Focus Add integers using number lines.

We can show the addition of whole numbers on a number line: $4 + 2 = 6$
Draw 2 arrows.



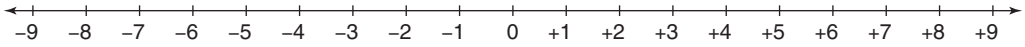
Or, begin at 4, and draw 1 arrow.



We can also show the addition of integers on a number line.

Explore

You will need copies of a number line.



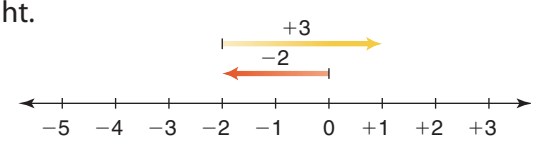
- Choose two different positive integers.
  Use a number line to add them.
  Write the addition equation.
- Repeat the activity for a positive integer and a negative integer.
- Repeat the activity for two different negative integers.
- What happens when you add $+2$ and -2?

Reflect & Share

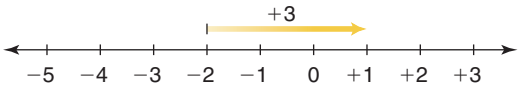
Compare your strategies for adding on a number line with those of your classmates.
Use coloured tiles to check the sums.
Why do you think integers such as $+2$ and -2 are called **opposite integers**?

Connect

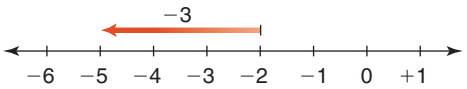
➤ To add a positive integer, move right (in the positive direction).
$(-2) + (+3)$
Start at 0.
Draw an arrow 2 units long, pointing left.
This arrow represents -2.
From -2, draw an arrow 3 units long, pointing right.
This arrow represents $+3$.
The arrow head is at $+1$.
So, $(-2) + (+3) = +1$



Notice that the first arrow ends at the first integer.
So, we could start at that integer,
and use only 1 arrow to find the sum.

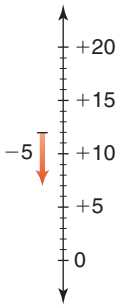


➤ To add a negative integer, move left
(in the negative direction).
$(-2) + (-3)$
Start at -2.
Draw an arrow 3 units long, pointing left.
This arrow represents -3.
The arrow head is at -5.
So, $(-2) + (-3) = -5$



We can use the same method to add integers
on a vertical number line.

➤ The temperature is 12°C. It falls 5°C.
Find the final temperature.
$(+12) + (-5)$
Start at $+12$.
Draw an arrow 5 units long, pointing down.
This arrow represents -5.
The arrow head is at $+7$.
So, $(+12) + (-5) = +7$
The final temperature is 7°C.



Example

Sandra and Joe buy and sell CDs at a flea market.
One day in August, they bought 3 CDs for $5 each.
They sold 2 CDs for $9 each.

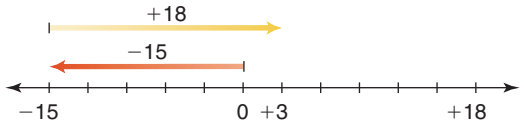
a) Write the expenses and income as integers.

b) Did Sandra and Joe make money or lose money that day in August? Explain.

A Solution

a) Expenses: $(-5) + (-5) + (-5) = -15$; they spent $15.
Income: $(+9) + (+9) = +18$; they made $18.

b) Draw a number line.
Add expenses and income.



$(-15) + (+18) = +3$
Since the sum of the expenses and income is positive, Sandra and Joe made money. They made $3.

Another Strategy
We could use coloured tiles.

Practice

1. Use a number line to represent each sum.

a) $(+1) + (+3)$ **b)** $(-1) + (+3)$ **c)** $(-3) + (+1)$ **d)** $(-1) + (-3)$
e) $(-3) + (-4)$ **f)** $(-3) + (+4)$ **g)** $(+3) + (-4)$ **h)** $(+3) + (+4)$

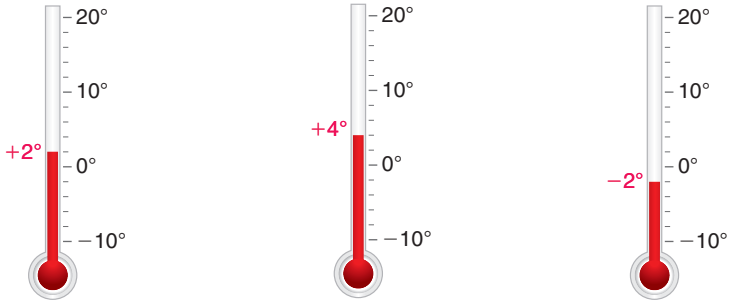
2. Use a number line to add.

a) $(+4) + (+2)$ **b)** $(+5) + (-3)$ **c)** $(-4) + (-2)$ **d)** $(-8) + (+2)$
e) $(-6) + (-7)$ **f)** $(+1) + (-6)$ **g)** $(-5) + (+2)$ **h)** $(+8) + (+4)$

3. **a)** Reverse the order of the integers in question 2, then add.
b) Compare your answers to the answers in question 2.
What do you notice?
c) Make a general statement about your observations.

4. Look at these thermometers. Find each temperature after:

a) it falls 4°C **b)** it falls 7°C **c)** it rises 6°C

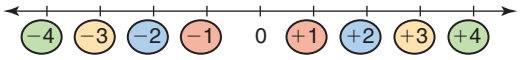


5. **a)** The temperature rises 7°C, then drops 2°C.
What is the overall change in temperature?

b) Adrian loses $4, then earns $8.
Did Adrian gain or lose overall?

c) The value of a stock went up $3, then down $2.
What was the final change in the value of the stock?

6. Opposite integers are the same distance from 0 but are on opposite sides of 0.



a) Write the opposite of each integer.

i) +2 **ii)** −5 **iii)** +6 **iv)** −8

b) Add each integer to its opposite in part a.

c) What do you notice about the sum of two opposite integers?

7. Use a number line. For each sentence below:

a) Write each number as an integer.

b) Write the addition equation.
Explain your answer in words.

i) You take 5 steps backward, then 10 steps backward.

ii) You withdraw $5, then deposit $8.

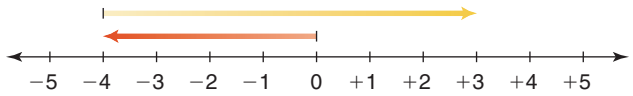
iii) A deep sea diver descends 8 m, then ascends 6 m.

iv) A person drives a snowmobile 4 km east, then 7 km west.

v) A person gains 6 kg, then loses 10 kg.

8. **a)** Write the addition equation modelled by each number line.
b) Describe a situation that each number line could represent.

i)



ii)

9. **Assessment Focus** Is each statement always true, sometimes true, or never true?
Use a number line to support your answers.
a) The sum of two opposite integers is 0.
b) The sum of two positive integers is negative.
c) The sum of two negative integers is negative.
d) The sum of a negative integer and a positive integer is negative.

10. **Take It Further** Add.
a) $(+4) + (+3) + (-6)$ **b)** $(-2) + (-4) + (+1)$
c) $(-5) + (+3) + (-4)$ **d)** $(+6) + (-8) + (+2)$

11. **Take It Further** The temperature in Calgary, Alberta, was -2°C.
A Chinook came through and the temperature rose 15°C.
At nightfall, it fell 7°C. What was the final temperature?
Support your answer with a drawing.

Reflect

Compare adding on a number line to adding with coloured tiles.
Which method do you prefer?
When might you need to use a different method?

Mid-Unit Review

LESSON

2.1 **1.** Use coloured tiles to model each integer in two different ways. Draw the tiles.

a) -5 **b)** 0

c) $+8$ **d)** -1

e) $+3$ **f)** -7

2. Suppose you have 8 red tiles. How many yellow tiles would you need to model $+3$? How do you know?

2.2 **3.** What sum does each set of tiles model? How do you know you are correct? Write the addition equations.

a) 6 yellow tiles and 1 red tile

b) 5 yellow tiles and 7 red tiles

c) 4 yellow tiles and 4 red tiles

4. Use coloured tiles to add. Draw pictures of the tiles you used.

a) $(+4) + (-1)$ **b)** $(-3) + (-2)$

c) $(-5) + (+1)$ **d)** $(+6) + (+3)$

e) $(-4) + (-8)$ **f)** $(+4) + (+8)$

2.3 **5.** Use a number line to add. Write the addition equations.

a) $(+3) + (+2)$ **b)** $(-5) + (-1)$

c) $(-10) + (+8)$ **d)** $(+6) + (-5)$

e) $(-8) + (+8)$ **f)** $(-5) + (+12)$

6. **a)** Add. $(+4) + (-5)$

b) Find 4 different pairs of integers that have the same sum as part a.

7. Write an addition equation for each situation.

a) Puja earned \$50, and spent \$20. How much did Puja then have?

b) The temperature is 5°C, then drops 10°C. What is the final temperature?

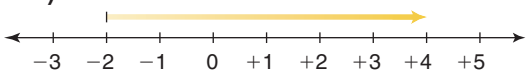
c) The population of a city was 124 000, then it dropped by 4000 people. What was the population then?

d) A plane was cruising at an altitude of 12 000 m, then dropped 1200 m. What was the cruising altitude then?

8. **a)** Write the addition equation modelled by each number line.

b) Describe a situation that each number line could represent.

i)



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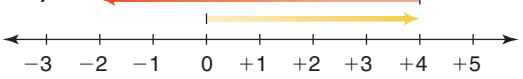


9. Each integer below is written as the sum of consecutive integers.

$(+5) = (+2) + (+3)$

$(+6) = (+1) + (+2) + (+3)$

Write each of these integers as the sum of consecutive integers.

a) $+10$ **b)** 0 **c)** $+2$

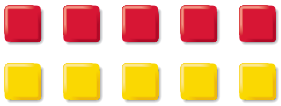
d) $+7$ **e)** $+4$ **f)** $+8$

2.4 Subtracting Integers with Tiles

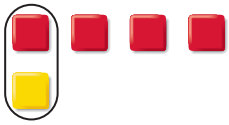
Focus Use coloured tiles to subtract integers.

To add integers, we combine groups of tiles.
To subtract integers, we do the reverse:
we remove tiles from a group.

Recall that equal numbers of
red and yellow tiles model 0.
For example, $+5$ and -5 form 5 zero pairs,
and $(-5) + (+5) = 0$



Adding a zero pair to a set of tiles does not change its value.
For example, $(-3) + 0 = -3$



Explore

You will need coloured tiles.
Use tiles to subtract.
Add zero pairs when you need to.
Sketch the tiles you used in each case.

- $(+5) - (+3)$
- $(+5) - (-3)$
- $(-3) - (+5)$
- $(-3) - (-5)$

Reflect & Share

Compare your results with those of another pair of classmates.
Explain why you may have drawn different sets of tiles, yet both may be correct.
When you subtracted, how did you know how many tiles to use
to model each integer? How did adding zero pairs help you?

Connect

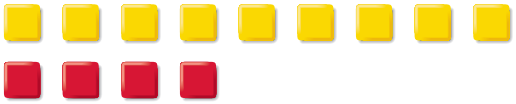
To use tiles to subtract integers, we model the first integer,
then take away the number of tiles indicated by the second integer.

We can use tiles to subtract: $(+5) - (+9)$

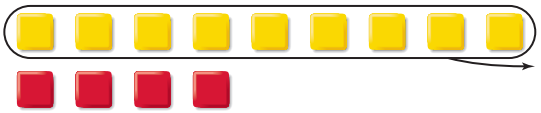
Model $+5$.

There are not enough tiles to take away $+9$.
To take away $+9$, we need 4 more yellow tiles.

We add zero pairs without changing the value.
Add 4 yellow tiles and 4 red tiles. They represent 0.



By adding 0, the integer the tiles represent has not changed.
Now take away the 9 yellow tiles.



Since 4 red tiles remain, we write: $(+5) - (+9) = -4$

This is a subtraction equation.

Example

Use tiles to subtract.

a) $(-2) - (-6)$ **b)** $(-6) - (+2)$ **c)** $(+2) - (-6)$

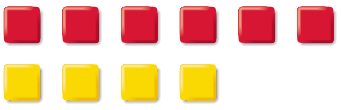
A Solution

a) $(-2) - (-6)$

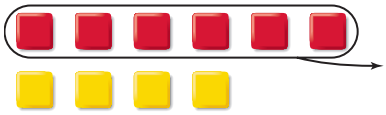
Model -2. 

There are not enough tiles to take away -6.
To take away -6, we need 4 more red tiles.

We add zero pairs without changing the value.
Add 4 red tiles and 4 yellow tiles.



Now take away 6 red tiles.



Since 4 yellow tiles remain, we write: $(-2) - (-6) = +4$

b) $(-6) - (+2)$

Model -6. 

There are no yellow tiles to take.
We need 2 yellow tiles to take away.

We add zero pairs.
Add 2 yellow tiles and 2 red tiles.

Now take away 2 yellow tiles.

Since 8 red tiles remain, we write: $(-6) - (+2) = -8$

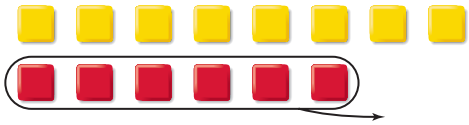
c) $(+2) - (-6)$

Model $+2$. 

There are no red tiles to take.
We need 6 red tiles to take away.

We add zero pairs.
Add 6 red tiles and 6 yellow tiles.

Now take away 6 red tiles.



Since 8 yellow tiles remain, we write: $(+2) - (-6) = +8$

Notice the results in the *Example*, parts b and c.
When we reverse the order in which we subtract two integers, the answer is the opposite integer.
$(-6) - (+2) = -8$
$(+2) - (-6) = +8$

Practice

1. Use tiles to subtract. Draw pictures of the tiles you used.

a) $(+7) - (+4)$ **b)** $(-2) - (-2)$ **c)** $(-9) - (-6)$

d) $(+4) - (+2)$ **e)** $(-8) - (-1)$ **f)** $(+3) - (+3)$

2. Use tiles to subtract.

a) $(-1) - (-4)$ **b)** $(+3) - (+8)$ **c)** $(-4) - (-11)$

d) $(+7) - (+8)$ **e)** $(-4) - (-6)$ **f)** $(+1) - (+10)$

3. Subtract.

a) $(-4) - (-1)$ **b)** $(+8) - (+3)$ **c)** $(-11) - (-4)$

d) $(+8) - (+7)$ **e)** $(-6) - (-4)$ **f)** $(+10) - (+1)$

4. Subtract. Write the subtraction equations.

a) $(+4) - (-7)$ **b)** $(-2) - (+8)$ **c)** $(-9) - (+5)$

d) $(+6) - (-8)$ **e)** $(-3) - (+6)$ **f)** $(-5) - (+7)$

5. Subtract.

a) $(+4) - (+5)$ **b)** $(-3) - (+5)$ **c)** $(-4) - (+3)$

d) $(-1) - (-8)$ **e)** $(+8) - (-2)$ **f)** $(+4) - (-7)$

6. Use questions 1 to 5 as models.
Write 3 integer subtraction questions.
Trade questions with a classmate.
Solve your classmate's questions.

7. **a)** Use coloured tiles to subtract each pair of integers.

i) $(+3) - (+1)$ and $(+1) - (+3)$

ii) $(-3) - (-2)$ and $(-2) - (-3)$

iii) $(+4) - (-3)$ and $(-3) - (+4)$

b) What do you notice about each pair of questions in part a?

8. $(+5) - (-2) = +7$
Predict the value of $(-2) - (+5)$.
Explain your prediction, then check it.

9. **Assessment Focus** Use integers.
Write a subtraction question that would give each answer.
How many questions can you write each time?

a) $+2$ **b)** -3 **c)** $+5$ **d)** -6

10. Which expression in each pair has the greater value?
Explain your reasoning.

a) i) $(+3) - (-1)$ **ii)** $(-3) - (+1)$

b) i) $(-4) - (-5)$ **ii)** $(+4) - (+5)$

11. Take It Further

a) Find two integers with a sum of -1 and a difference of $+5$.

b) Create and solve a similar integer question.

12. Take It Further Copy and complete.

a) $(+4) - \square = +3$

b) $(+3) - \square = -1$

c) $\square - (+1) = +4$

13. Take It Further Evaluate.

a) $(+4) + (+1) - (+3)$

b) $(+1) - (+2) - (-1)$

c) $(-3) - (+1) + (+4)$

d) $(-2) - (-4) + (-1)$

e) $(+2) - (+1) - (+4)$

f) $(+1) - (+2) + (+1)$

14. Take It Further Here is a magic square.

a) Subtract $+4$ from each entry.
Is it still a magic square? Why?

b) Subtract -1 from each entry.
Is it still a magic square? Why?

0	$+5$	-2
-1	$+1$	$+3$
$+4$	-3	$+2$

Reflect

Here are 4 types of subtraction questions:

- (negative integer) $-$ (negative integer)
- (negative integer) $-$ (positive integer)
- (positive integer) $-$ (positive integer)
- (positive integer) $-$ (negative integer)

Write a question for each type of subtraction.
Show how you use tiles to solve each question.