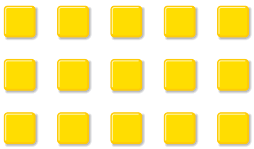
2.2 Developing Rules to Multiply Integers

Focus Use patterns to develop rules for multiplying integers.

We can write the number of tiles in this array in two ways.

As a sum:

$(+5) + (+5) + (+5) = +15$



As a product:

$(+3) \times (+5) = +15$

How can you use integers to write the number of tiles in this array in two ways?

Investigate

Work with a partner.

Your teacher will give you a large copy of this multiplication table.

Fill in the products that you know best.
Use any patterns you see to help you complete the table.

Second Number

First Number

×	−5	−4	−3	−2	−1	0	+1	+2	+3	+4	+5
−5											
−4											
−3											
−2											
−1											
0											
+1											
+2											
+3											
+4											
+5											

Reflect & Share

Compare your completed table with that of another pair of classmates.
Explain a strategy you could use to multiply any negative integer by any positive integer.
Explain a strategy you could use to multiply any two negative integers.

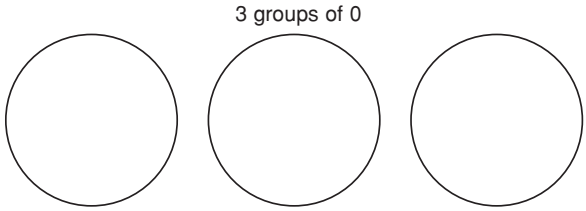
Connect

These properties of whole numbers are also properties of integers.

Multiplying by 0 (Zero property)

$3 \times 0 = 0$ and $0 \times 3 = 0$

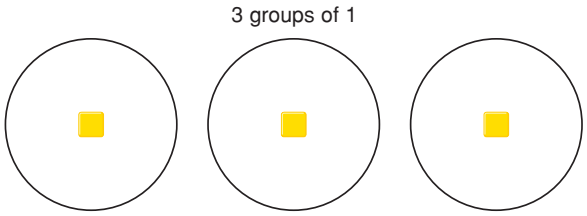
So, $(-3) \times 0 = 0$ and $0 \times (-3) = 0$



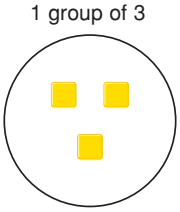
Multiplying by 1 (Multiplicative Identity)

$3 \times 1 = 3$ and $1 \times 3 = 3$

So, $(-3) \times (+1) = -3$ and $(+1) \times (-3) = -3$



Since multiplying by 1 does not change the identity of a number, we call 1 the *multiplicative identity*.



Commutative Property

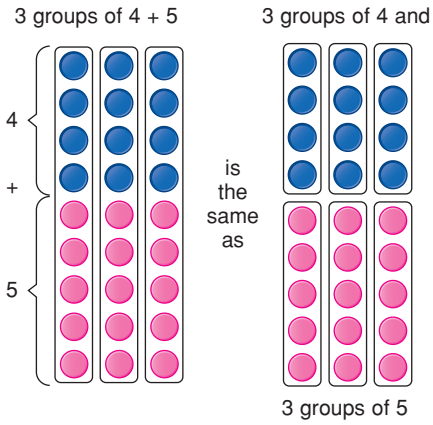
$3 \times 4 = 12$ and $4 \times 3 = 12$

So, $(-3) \times (+4) = -12$ and $(+4) \times (-3) = -12$

Distributive Property

$$\begin{aligned} 3 \times (4 + 5) &= 3 \times 4 + 3 \times 5 \\ &= 12 + 15 \\ &= 27 \end{aligned}$$

So, $$\begin{aligned} (+3) \times [(-4) + (-5)] &= [(+3) \times (-4)] + [(+3) \times (-5)] \\ &= (-12) + (-15) \\ &= -27 \end{aligned}$$



Example 1

Find each product.

a) $(-9) \times (+4)$ **b)** $(-4) \times (-9)$ **c)** $(+4) \times (+9)$

A Solution

a) Multiply the numbers as if they were positive.
$9 \times 4 = 36$
The integers have opposite signs, so the product is negative.
So, $(-9) \times (+4) = -36$

b) The integers have the same sign, so the product is positive.
So, $(-4) \times (-9) = +36$

c) The integers have the same sign, so the product is positive.
So, $(+4) \times (+9) = +36$

Example 2

Find the product: $(+20) \times (-36)$

A Solution

$$\begin{aligned}(+20) \times (-36) &= (+20) \times [(-30) + (-6)] \\ &= [(+20) \times (-30)] + [(+20) \times (-6)] \\ &= (-600) + (-120) \\ &= -720\end{aligned}$$

So, $(+20) \times (-36) = -720$

Write –36 in expanded form.
Use the distributive property.

Example 3

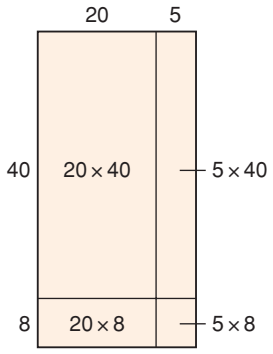
Find the product: $(-25) \times (-48)$

A Solution

Multiply the numbers as if they were positive: 25×48
Use a rectangle model.

$$\begin{aligned}(25) \times (48) &= (20 \times 40) + (5 \times 40) + (20 \times 8) + (5 \times 8) \\ &= 800 + 200 + 160 + 40 \\ &= 1200\end{aligned}$$

The integers have the same sign, so the product is positive.
So, $(-25) \times (-48) = +1200$



When we write the product of integers, we do not need to write the multiplication sign. That is, we may write $(-8) \times (-9)$ as $(-8)(-9)$.

Discuss the ideas

1. Think about the work of this lesson and the previous lesson. What is the sign of the product when you multiply 2 integers:
   - if both integers are positive?
   - if one integer is positive and the other integer is negative?
   - if both integers are negative?
2. Explain your strategy to multiply 2 integers.

Practice

Check

3. Will each product be positive or negative? How do you know?
   a) $(-6) \times (+2)$ b) $(+6) \times (+4)$
   c) $(+4) \times (-2)$ d) $(-7) \times (-3)$

4. Find each product.
   a) $(+8)(-3)$ b) $(-5)(-4)$
   c) $(-3)(+9)$ d) $(+7)(-6)$
   e) $(+10)(-3)$ f) $(-7)(-6)$
   g) $(0)(-8)$ h) $(+10)(-1)$
   i) $(-7)(-8)$ j) $(+9)(-9)$

5. a) Find the product of each pair of integers.
   i) $(+3)(-7)$ and $(-7)(+3)$
   ii) $(+4)(+8)$ and $(+8)(+4)$
   iii) $(-5)(-9)$ and $(-9)(-5)$
   iv) $(-6)(+10)$ and $(+10)(-6)$
   b) Use the results of part a. Does the order in which integers are multiplied affect the product? Explain.

6. Find each product.
   a) $(+20) \times (+15)$ b) $(-30) \times (-26)$
   c) $(+50) \times (-32)$ d) $(-40) \times (+21)$
   e) $(-60) \times (+13)$ f) $(+80) \times (-33)$
   g) $(+70) \times (+47)$ h) $(-90) \times (-52)$

Apply

7. Find each product.
   a) $(+25) \times (-12)$ b) $(-45) \times (+21)$
   c) $(-34) \times (-16)$ d) $(-37) \times (+18)$
   e) $(+17)(+13)$ f) $(+84)(-36)$
   g) $(-51)(-25)$ h) $(+29)(+23)$

8. Copy each equation. Replace $\square$ with an integer to make the equation true.
   a) $(+5) \times \square = +20$
   b) $\square \times (-9) = +27$
   c) $(-9) \times \square = -54$
   d) $\square \times (-3) = +18$
   e) $\square \times (+5) = -20$
   f) $\square \times (-12) = +144$
   g) $\square \times (-6) = +180$
   h) $\square \times (-4) = +24$

9. Write the next 3 terms in each pattern. Then write the pattern rule.

a) +1, +2, +4, +8, …

b) +1, –6, +36, –216, …

c) –1, +3, –9, +27, …

d) –4, +4, –4, +4, …

10. Gaston withdrew $26 from his bank account each week for 17 weeks. Use integers to find the total amount Gaston withdrew over the 17 weeks. Show your work.

11. **Assessment Focus** Use the integers: –5, +9, –8, +4, –2

a) Which two integers have the greatest product?

b) Which two integers have the least product?

c) Provide a convincing argument that your answers to parts a and b are correct.

12. **a)** Find each product. Then use a calculator to extend the pattern 4 more rows.

i) (–2)(–3)

ii) (–2)(–3)(–4)

iii) (–2)(–3)(–4)(–5)

iv) (–2)(–3)(–4)(–5)(–6)

b) Use the results in part a.

i) What is the sign of a product when it has an even number of negative factors? Explain.

ii) What is the sign of a product when it has an odd number of negative factors? Explain.

c) Investigate what happens when a product has positive and negative factors. Do the rules in part b still apply? Explain.

13. Amelie was doing a math question. The answer she got did not match the answer in the answer key. So, she asked a friend to look at her work.

$$
\begin{aligned}
&(+60) \times (-18) \\
&\quad = (+60) \times [(-20) + (+2)] \\
&\quad = [(+60) \times (-20)] + [(+60) \times (+2)] \\
&\quad = (+1200) + (+120) \\
&\quad = +1320
\end{aligned}
$$

a) What was Amelie's error?

b) Correct Amelie's error. What is the correct answer?

14. Gavin used the expression $(+15) \times (-8)$ to solve a word problem. What might the word problem have been? Solve the problem.

15. Explain why an integer multiplied by itself can never result in a negative product.

16. Bridget used the expression $(-12) \times (+7)$ to solve a word problem. What might the word problem have been? Solve the problem.

17. Write –36 as the product of two or more integer factors. Do this as many different ways as you can. Show your work.

18. The product of two integers is –144. The sum of the integers is –7. What are the two integers?

19. **Take It Further** When you multiply two natural numbers, the product is never less than either of the two numbers. Is the same statement true for the product of any two integers? Investigate, then write what you find out.

Natural numbers are 1, 2, 3, 4, …, and so on.

20. **Take It Further** The product of two integers is between +160 and +200. One integer is between –20 and –40.

a) What is the greatest possible value of the other integer?

b) What is the least possible value of the other integer?

21. **Take It Further** How can you use the rules for multiplying two integers to multiply more than two integers? Use an example to illustrate your strategy.

Reflect

Suppose your friend missed this lesson. Explain to her how to multiply two integers. Use examples in your explanation.

What's My Product?

Game

In this game, a black card represents a positive integer; for example, the 5 of spades is +5.
A red card represents a negative integer; for example, the 6 of hearts is –6.

HOW TO PLAY

1. Remove the face cards (Jacks, Queens, Kings) from the deck. Use the remaining cards. An ace is 1.

2. Shuffle the cards. Place them face down in a pile. Players take turns to turn over the top 2 cards, then find the product. The person with the greater product keeps all 4 cards.

3. If there is a tie, each player turns over 2 more cards and the player with the greater product keeps all 8 cards.

4. Play continues until all cards have been used. The winner is the player with more cards.

YOU WILL NEED

A standard deck of playing cards

NUMBER OF PLAYERS

2

GOAL OF THE GAME

To have more cards