3.1 Fractions to Decimals

Focus Use patterns to convert between decimals and fractions.

Numbers can be written in both fraction and decimal form.
For example, 3 can be written as $\frac{3}{1}$ and 3.0.
A fraction illustrates division;
that is, $\frac{1}{10}$ means $1 \div 10$.
Recall that $\frac{1}{10}$ is 0.1 in decimal form.
$\frac{3}{100}$ is 0.03 in decimal form.
$\frac{45}{1000}$ is 0.045 in decimal form.
Here are some more fractions and decimals you learned in earlier grades.

Fraction	$\frac{7}{10}$	$\frac{1}{100}$	$\frac{19}{100}$	$\frac{1}{1000}$	$\frac{23}{1000}$	$\frac{471}{1000}$
Decimal	0.7	0.01	0.19	0.001	0.023	0.471

Explore

You will need a calculator.

➤ Use a calculator.
Write each fraction as a decimal: $\frac{1}{11}, \frac{2}{11}, \frac{3}{11}, \frac{4}{11}$
What patterns do you see?
Use your patterns to predict the decimal forms of these fractions:
$\frac{5}{11}, \frac{6}{11}, \frac{7}{11}, \frac{8}{11}, \frac{9}{11}, \frac{10}{11}$
Use a calculator to check your predictions.

➤ Use a calculator.
Write each fraction as a decimal: $\frac{1}{9}, \frac{2}{9}, \frac{3}{9}$
What patterns do you see?
Use your patterns to predict the fraction form of these decimals:
0.777 777 777… 0.888 888 888…
Check your predictions.
What do you notice about the last digit in the calculator display?

Reflect & Share

Compare your patterns, decimals, and fractions with those of another pair of classmates. How did you use patterns to make predictions?

Connect

➤ Decimals, such as 0.1 and 0.25, are **terminating decimals**.
Each decimal has a definite number of decimal places.

➤ Decimals, such as 0.333 333 333…; 0.454 545 454…; 0.811 111 111…
are **repeating decimals**.
Some digits in each repeating decimal repeat forever.
We draw a bar over the digits that repeat.
For example, $\frac{4}{33} = 4 \div 33 = 0.121\ 212\ 121\ldots$, which is written as $0.\overline{12}$
$\frac{73}{90} = 73 \div 90 = 0.811\ 111\ 111\ldots$, which is written as $0.8\overline{1}$

➤ Patterns sometimes occur when we write fractions in decimal form.
For example,
$\frac{1}{99} = 0.\overline{01}$ $\frac{2}{99} = 0.\overline{02}$ $\frac{15}{99} = 0.\overline{15}$ $\frac{43}{99} = 0.\overline{43}$
For fractions with denominator 99, the digits in the numerator of the fraction are the repeating digits in the decimal.
We can use this pattern to make predictions.
To write $0.\overline{67}$ as a fraction, write the repeating digits, 67, as the numerator of a fraction with denominator 99.
$0.\overline{67} = \frac{67}{99}$
Similarly, $0.\overline{7} = 0.\overline{77} = \frac{77}{99} = \frac{7}{9}$

Example

a) Write each fraction as a decimal.
b) Sort the fractions as representing repeating or terminating decimals:
$\frac{13}{200}, \frac{1}{5}, \frac{11}{20}, \frac{3}{7}$

A Solution

a) Try to write each fraction with denominator 10, 100, or 1000.

$\frac{13}{200} \xrightarrow{\times 5} \frac{65}{1000}$, or 0.065

$\frac{1}{5} \xrightarrow{\times 2} \frac{2}{10}$, or 0.2

$\frac{11}{20} \xrightarrow{\times 5} \frac{55}{100}$, or 0.55

$\frac{3}{7}$ cannot be written as a fraction with denominator 10, 100, or 1000.

Use a calculator.

$\frac{3}{7} = 3 \div 7 = 0.428\,571\,429$

This appears to be a terminating decimal.

We use long division to check.

Since we are dividing by 7, the remainders must be less than 7.

Since we get a remainder that occurred before, the division repeats.

So, $\frac{3}{7} = 0.\overline{428\,571}$

```
   0.4285714
7)3.00000000
  28
  20
  14
   60
   56
    40
    35
     50
     49
      10
       7
       30
       28
        20
```

The calculator rounds the decimal to fit the display:

$\frac{3}{7} = 0.428\,571\,428\,571\ldots$

This is the last digit in the display.

Since this digit is 5, the calculator adds 1 to the preceding digit.

So, the calculator displays an approximate decimal value:

$\frac{3}{7} \doteq 0.428\,571\,429$

b) Since 0.065, 0.2, and 0.55 terminate, $\frac{13}{200}$, $\frac{1}{5}$, and $\frac{11}{20}$ represent terminating decimals.
Since $0.\overline{428\,571}$ repeats, $\frac{3}{7}$ represents a repeating decimal.

Practice

Use a calculator when you need to.

1. **a)** Write each fraction as a decimal.

i) $\frac{2}{3}$ **ii)** $\frac{3}{4}$ **iii)** $\frac{4}{5}$ **iv)** $\frac{5}{6}$ **v)** $\frac{6}{7}$

b) Identify each decimal as terminating or repeating.

2. Write each decimal as a fraction.

a) 0.9 **b)** 0.26 **c)** 0.45 **d)** 0.01 **e)** 0.125

3. **a)** Write each fraction as a decimal.

i) $\frac{1}{27}$ **ii)** $\frac{2}{27}$ **iii)** $\frac{3}{27}$

b) Describe the pattern in your answers to part a.

c) Use your pattern to predict the decimal form of each fraction.

i) $\frac{4}{27}$ **ii)** $\frac{5}{27}$ **iii)** $\frac{8}{27}$

4. For each fraction, write an equivalent fraction with denominator 10, 100, or 1000.
Then, write the fraction as a decimal.

a) $\frac{2}{5}$ **b)** $\frac{1}{4}$ **c)** $\frac{13}{25}$ **d)** $\frac{19}{50}$ **e)** $\frac{37}{500}$

5. Write each decimal as a fraction in simplest form.

a) $0.\overline{6}$ **b)** $0.\overline{5}$ **c)** $0.\overline{41}$ **d)** $0.\overline{16}$

6. Write each fraction as a decimal.

a) $\frac{4}{7}$ **b)** $\frac{4}{9}$ **c)** $\frac{6}{11}$ **d)** $\frac{7}{13}$

7. Write $\frac{5}{17}$ as a decimal.
The calculator display is not long enough to show the repeating digits.
How could you find the repeating digits?

8. Write $\frac{1}{5}$ as a decimal.
Use this decimal to write each number below as a decimal.

a) $\frac{4}{5}$ **b)** $\frac{7}{5}$ **c)** $\frac{9}{5}$ **d)** $\frac{11}{5}$

9. **a)** Write each fraction as a decimal.

i) $\frac{1}{999}$ **ii)** $\frac{2}{999}$ **iii)** $\frac{54}{999}$ **iv)** $\frac{113}{999}$

b) Describe the pattern in your answers to part a.

c) Use your pattern to predict the fraction form of each decimal.

i) $0.\overline{004}$ **ii)** $0.\overline{089}$ **iii)** $0.\overline{201}$ **iv)** $0.\overline{326}$

10. Match each set of decimals and fractions.
Explain how you know.

a) $\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \frac{5}{3}$ **i)** 0.125, 0.25, 0.375, 0.5, 0.625

b) $\frac{1}{8}, \frac{2}{8}, \frac{3}{8}, \frac{4}{8}, \frac{5}{8}$ **ii)** $0.1\overline{6}$, $0.\overline{3}$, 0.5, $0.\overline{6}$, $0.8\overline{3}$

c) $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{5}{5}$ **iii)** $0.\overline{3}$, $0.\overline{6}$, 1.0, $1.\overline{3}$, $1.\overline{6}$

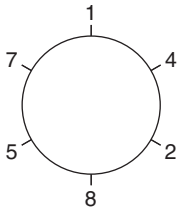
d) $\frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}$ **iv)** 0.2, 0.4, 0.6, 0.8, 1.0

11. **Assessment Focus** Here is the Fibonacci sequence:
1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, …
We can write consecutive terms as fractions:
$\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}$, and so on

a) Write each fraction above as a decimal.
What do you notice about the trend in the decimals?

b) Continue to write consecutive terms as decimals.
Write about what you find out.

12. **a)** Write $\frac{1}{7}$ as a repeating decimal.
How many digits repeat?
These repeating digits are shown around the circle at the right.



b) Write the fractions $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}$, and $\frac{6}{7}$ in decimal form.
What patterns do you see?
Explain how the circle of digits can help you write these fractions as decimals.

13. **Take It Further**

a) Write each fraction as a decimal.
Identify the decimals as repeating or terminating.
i) $\frac{7}{8}$ **ii)** $\frac{5}{18}$ **iii)** $\frac{3}{10}$ **iv)** $\frac{8}{27}$ **v)** $\frac{4}{25}$

b) Write the denominator of each fraction in part a as a product of prime factors.

c) What do you notice about the prime factors of the denominators of the terminating decimals?
The repeating decimals?

d) Use your answers to part c.
Predict which of these fractions can be written as terminating decimals.
i) $\frac{7}{15}$ **ii)** $\frac{13}{40}$ **iii)** $\frac{5}{81}$ **iv)** $\frac{9}{16}$

A prime number has exactly two factors, itself and 1. We can write 12 as a product of prime factors: $2 \times 2 \times 3$

Reflect

Sometimes it is hard to figure out if a fraction can be written as a terminating decimal or a repeating decimal.
What can you do if you are stuck?

3.2 Comparing and Ordering Fractions and Decimals

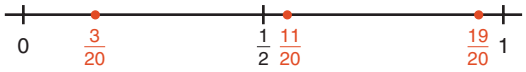
Focus Use benchmarks, place value, and equivalent fractions to compare and order fractions and decimals.

Recall how to use the benchmarks 0, $\frac{1}{2}$, and 1 to compare fractions.

For example, $\frac{3}{20}$ is close to 0 because the numerator is much less than the denominator.

$\frac{11}{20}$ is close to $\frac{1}{2}$ because the numerator is about $\frac{1}{2}$ the denominator.

$\frac{19}{20}$ is close to 1 because the numerator and denominator are close in value.



Explore

Use any materials to help.

Dusan, Sasha, and Kimberley sold chocolate bars as a fund-raiser for their choir.

The bars were packaged in cartons, but sold individually.

Dusan sold $2\frac{2}{3}$ cartons. Sasha sold $\frac{5}{2}$ cartons. Kimberley sold 2.25 cartons.

Who sold the most chocolate bars?

Reflect & Share

Share your solution with another pair of classmates.

How did you decide which number was greatest?

Did you use any materials to help? How did they help?

Try to find a way to compare the numbers without using materials.

Connect

Any fraction greater than 1 can be written as a mixed number.

The benchmarks 0, $\frac{1}{2}$, and 1 can be used to compare the fraction parts of mixed numbers.

We can use benchmarks on a number line to order these numbers: $\frac{2}{11}, 2\frac{3}{8}, 1\frac{1}{16}, \frac{14}{9}, \frac{14}{15}$

$\frac{2}{11}$ is close to 0.

Since $\frac{3}{8}$ is close to $\frac{1}{2}$, but less than $\frac{1}{2}$,

$2\frac{3}{8}$ is close to $2\frac{1}{2}$, but less than $2\frac{1}{2}$.

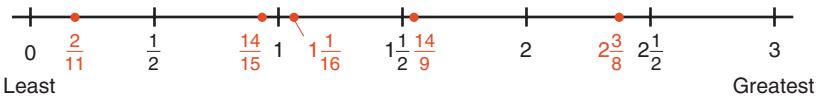
$1\frac{1}{16}$ is close to 1, but greater than 1.

$\frac{14}{9} = \frac{9}{9} + \frac{5}{9} = 1\frac{5}{9}$

$1\frac{5}{9}$ is close to $1\frac{1}{2}$, but greater than $1\frac{1}{2}$.

$\frac{14}{15}$ is close to 1, but less than 1.

Place the fractions on a number line.



The numbers in order from greatest to least are: $2\frac{3}{8}, \frac{14}{9}, 1\frac{1}{16}, \frac{14}{15}, \frac{2}{11}$

We can also use equivalent fractions to order fractions.

Example

a) Write these numbers in order from least to greatest: $\frac{7}{8}, \frac{9}{8}, 1\frac{1}{4}, 0.75$

b) Write a fraction between $\frac{9}{8}$ and $1\frac{1}{4}$.

A Solution

a) Write equivalent fractions with like denominators, then compare the numerators.

First write the decimal as a fraction: $0.75 = \frac{75}{100} = \frac{3}{4}$

Compare: $\frac{7}{8}, \frac{9}{8}, 1\frac{1}{4}, \frac{3}{4}$

Since 8 is a multiple of 4, use 8 as a common denominator.

$1\frac{1}{4} = \frac{4}{4} + \frac{1}{4}$

$= \frac{5}{4}$

$\frac{3}{4} = \frac{6}{8}$ (× 2)

$\frac{5}{4} = \frac{10}{8}$ (× 2)

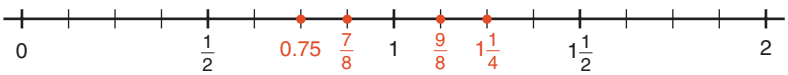
Each fraction now has denominator 8: $\frac{7}{8}, \frac{9}{8}, \frac{10}{8}, \frac{6}{8}$

Compare the numerators: $6 < 7 < 9 < 10$

So, $\frac{6}{8} < \frac{7}{8} < \frac{9}{8} < \frac{10}{8}$

So, $0.75 < \frac{7}{8} < \frac{9}{8} < 1\frac{1}{4}$

We can verify this order by placing the numbers on a number line.



b) Use the equivalent fraction for $1\frac{1}{4}$ with denominator 8 from part a: $\frac{10}{8}$

Find a fraction between $\frac{9}{8}$ and $\frac{10}{8}$.

The numerators are consecutive whole numbers. There are no whole numbers between 9 and 10. Multiply the numerator and denominator of both fractions by the same number to get equivalent fractions.

Choose 2:

$\frac{9}{8} \xrightarrow{\times 2} \frac{18}{16}$ $\qquad$ $\frac{10}{8} \xrightarrow{\times 2} \frac{20}{16}$

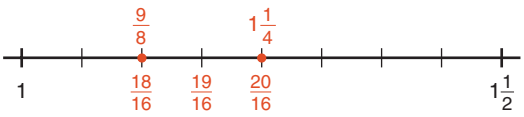
Look at the numerators.

19 is between 18 and 20,

so $\frac{19}{16}$ is between $\frac{18}{16}$ and $\frac{20}{16}$.

So, $\frac{19}{16}$, or $1\frac{3}{16}$, is between $\frac{9}{8}$ and $1\frac{1}{4}$.

We can verify this using a number line.



Another Solution

We can also use place value to order decimals.

a) Write each number as a decimal.

$\frac{7}{8} = 0.875$ $\qquad$ $\frac{9}{8} = 1.125$ $\qquad$ $1\frac{1}{4} = 1.25$ $\qquad$ 0.75

Write each decimal in a place-value chart.

Compare the ones.

Two numbers have 1 one and two numbers have 0 ones.

Ones	Tenths	Hundredths	Thousandths
0	8	7	5
1	1	2	5
1	2	5	0
0	7	5	0

Look at the decimals with 0 ones: 0.**8**75, 0.**7**50

Compare the tenths: 7 tenths is less than 8 tenths, so 0.750 < 0.875

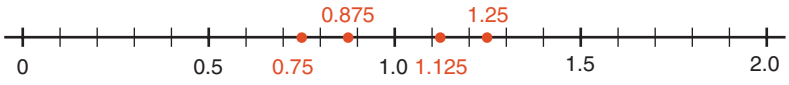
Look at the decimals with 1 one: 1.**1**25 and 1.**2**50

Compare the tenths: 1 tenth is less than 2 tenths, so 1.125 < 1.250

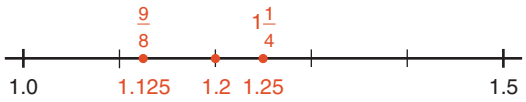
The numbers in order from least to greatest are: 0.750, 0.875, 1.125, 1.250

So, $0.75 < \frac{7}{8} < \frac{9}{8} < 1\frac{1}{4}$

We can verify this using a number line.



b) $\frac{9}{8} = 1.125$ $\quad 1\frac{1}{4} = 1.25$



Use the number line above.

1.2 lies between 1.125 and 1.25.

Write 1.2 as a fraction.

1.2 is $1\frac{2}{10}$, or $1\frac{1}{5}$.

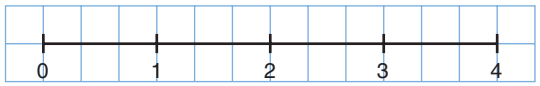
So, $1\frac{1}{5}$, or $\frac{6}{5}$, lies between $\frac{9}{8}$ and $1\frac{1}{4}$.

There are many other possible fractions between $\frac{9}{8}$ and $1\frac{1}{4}$.

Practice

1. Write 5 different fractions with like denominators.
Draw a number line, then order the fractions on the line.
Explain your strategy.

2. Use 1-cm grid paper.
Draw a 12-cm number line like the one shown.
Use the number line to order these numbers from greatest to least.
$2\frac{1}{2}, \frac{11}{3}, 2\frac{5}{6}$



3. Use benchmarks and a number line to order each set of numbers from least to greatest.

a) $\frac{7}{6}, \frac{15}{12}, 1\frac{2}{9}, 1$ **b)** $1\frac{3}{4}, \frac{7}{3}, \frac{7}{6}, 2$ **c)** $\frac{7}{4}, \frac{15}{10}, \frac{11}{5}, 2$ **d)** $\frac{10}{4}, 2\frac{1}{3}, \frac{9}{2}, 3$

4. Use equivalent fractions.
Order each set of numbers from greatest to least.
Verify by writing each fraction as a decimal.

a) $3\frac{1}{2}, \frac{13}{4}, 3\frac{1}{8}$ **b)** $\frac{5}{6}, \frac{2}{3}, 1\frac{1}{12}, \frac{9}{12}$ **c)** $1\frac{2}{5}, \frac{4}{3}, \frac{3}{2}$

5. Use place value.
Order each set of numbers from least to greatest.
Verify by using a number line.

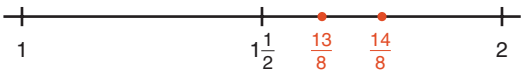
a) $\frac{7}{4}, 1.6, 1\frac{4}{5}, 1.25, 1$ **b)** $2\frac{5}{8}, 1.875, 2\frac{3}{4}, \frac{5}{2}, 2$

6. **a)** Use any method. Order these numbers from greatest to least.
Explain the method you used.
$\frac{17}{5}, 3.2, 2.8, 3\frac{1}{4}, \frac{21}{7}, 2$

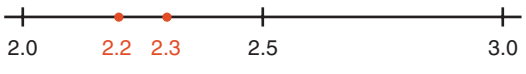
b) Use a different method. Verify your answer in part a.

7. Find a number between the two numbers represented by each pair of dots.

a)



b)

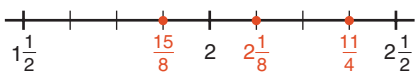


8. Find a number between each pair of numbers.

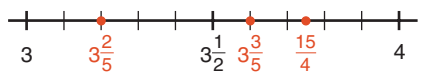
a) $\frac{5}{7}, \frac{6}{7}$ **b)** $1\frac{2}{5}, \frac{8}{5}$ **c)** $1.3, 1\frac{2}{5}$ **d)** $0.5, 0.6$

9. Identify the number that has been placed incorrectly.
Explain how you know.

a)



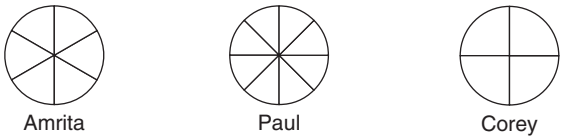
b)



10. In each set, identify the number that is not in the correct order.
Show where it should go. Explain your work.

a) $\frac{29}{5}, 6\frac{2}{10}, 6.25, 6\frac{2}{20}$ **b)** $1\frac{7}{16}, 1\frac{3}{8}, \frac{3}{2}, 1.2, \frac{3}{4}$

11. **Assessment Focus** Amrita, Paul, and Corey baked pizzas for the fund-raising sale.
The students cut their pizzas into different sized slices.



Amrita sold $\frac{11}{6}$ pizzas. Paul sold 1.875 pizzas. Corey sold $\frac{9}{4}$ pizzas.

a) Use a number line to order the numbers of pizzas sold from least to greatest.

b) Who sold the most pizzas? The fewest pizzas?

c) Use a different method. Verify your answers in part b.

d) Alison sold $2\frac{1}{5}$ pizzas. Where does this fraction fit in part a?

Reflect

Describe 3 ways to compare and order fractions and decimals.
Give an example of when you would use each method.
Which way do you prefer? Why?