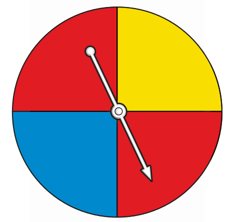
7.3 Probability of Independent Events

Focus Develop and apply a rule to determine the probability of two independent events.

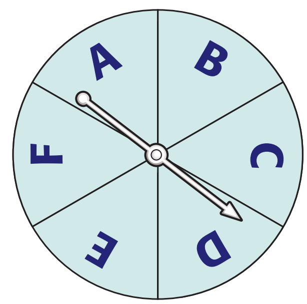
James played a game.
He spun the pointer on this spinner and tossed the coin.
What is the probability that the pointer lands on red?
Does the spinner result affect the coin toss result?

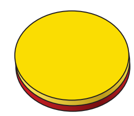


Investigate

Work with a partner.

➤ Use a tree diagram.
List the possible outcomes of spinning the pointer on this spinner and tossing the two-coloured counter.
What is the probability of each event?
- landing on F
- tossing red
- landing on F and tossing red





➤ Each of Kelsey and Sidney has a standard deck of 52 playing cards.
Each student turns over a card, then the students compare suits.
Make a table to list the possible outcomes.
What is the probability of each event?
- Kelsey turns over a spade.
- Sidney turns over a heart.
- Kelsey turns over a spade and Sidney turns over a heart.

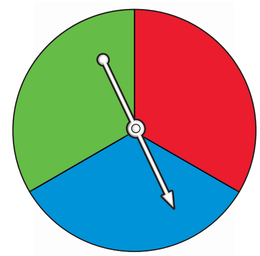
➤ In each situation above, how does the probability of each individual event relate to the probability of the combined events?
Write a rule to find the probability of two independent events.
Use your rule to find the probability of tossing heads on a coin and drawing a red tile from a bag that contains 2 red tiles and 3 green tiles.
Use a tree diagram to check your probability.

Reflect & Share

Compare your results and rule with those of another pair of classmates.
Did you write the same rule? If not, do both rules work?
Does the order in which the events are performed matter? Why or why not?

Connect

Two events are **independent events** when one event does not affect the other event.



The pointer on this spinner is spun twice. Landing on red and landing on blue are examples of two independent events.

Use a table to find the probability of landing on red twice.

There are 9 possible outcomes:
RR, RB, RG, BR, BB, BG, GR, GB, GG
Only one outcome is RR.
So, the probability of landing on red twice is $\frac{1}{9}$.

First Spin

Second Spin		**R**	**B**	**G**
	R	RR	RB	RG
	B	BR	BB	BG
	G	GR	GB	GG

The probability of landing on red on the first spin is $\frac{1}{3}$.

The probability of landing on red on the second spin is $\frac{1}{3}$.
Note that: $\frac{1}{9} = \frac{1}{3} \times \frac{1}{3}$

probability of landing on red twice = probability of landing on red on the first spin × probability of landing on red on the second spin

This illustrates the rule below for two independent events.

Suppose the probability of event A is written as P(A).
The probability of event B is written as P(B).
Then, the probability that both A and B occur is written as P(A and B).
If A and B are independent events, then: P(A and B) $=$ P(A) $\times$ P(B)

Example 1

A coin is tossed and a regular tetrahedron labelled 5, 6, 7, and 8 is rolled.

a) Find the probability of tossing heads and rolling an 8.
b) Find the probability of tossing heads or tails and rolling an even number.

Use a tree diagram to verify your answers.

A Solution

Since the outcome of tossing the coin does not depend on the outcome of rolling the tetrahedron, the events are independent.

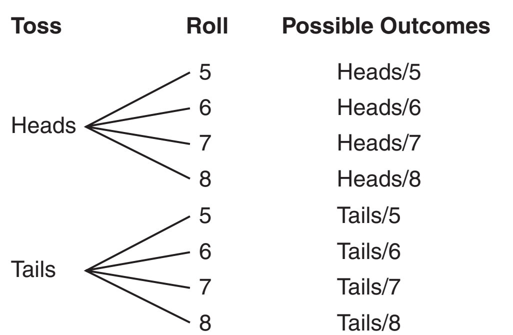
a) When the coin is tossed, there are 2 possible outcomes.
One outcome is heads.
So, $P(\text{heads}) = \frac{1}{2}$
When the tetrahedron is rolled, there are 4 possible outcomes.
One outcome is an 8.
So, $P(8) = \frac{1}{4}$

$$\begin{aligned} P(\text{heads and } 8) &= P(\text{heads}) \times P(8) \\ &= \frac{1}{2} \times \frac{1}{4} \\ &= \frac{1}{8} \end{aligned}$$

b) When the coin is tossed, there are 2 possible outcomes.
Two outcomes are heads or tails.
So, $P(\text{heads or tails}) = \frac{2}{2} = 1$
When the tetrahedron is rolled, there are 4 possible outcomes.
Two outcomes are even numbers: 6 and 8
So, $P(\text{even number}) = \frac{2}{4} = \frac{1}{2}$

$$\begin{aligned} P(\text{heads or tails and even number}) &= P(\text{heads or tails}) \times P(\text{even number}) \\ &= 1 \times \frac{1}{2} \\ &= \frac{1}{2} \end{aligned}$$

Use a tree diagram to check your answers.



There are 8 possible outcomes.
One outcome is heads/8.
So the probability of tossing heads and rolling 8 is $\frac{1}{8}$.
Four outcomes have heads or tails and an even number: heads/6, heads/8, tails/6, tails/8
So, the probability of tossing heads or tails and rolling an even number is $\frac{4}{8}$, or $\frac{1}{2}$.

Example 2

The pocket of a golf bag contains 9 white tees, 7 red tees, and 4 blue tees.
The golfer removes 1 tee from her bag without looking, notes the colour, then returns the tee to the pocket.
The process is repeated.
Find the probability of each event.

a) Both tees are red.

b) The first tee is not red and the second tee is blue.

A Solution

Since the first tee is returned to the pocket, the events are independent.

a) There are 20 tees in the pocket.

$$P(\text{red}) = \frac{7}{20}$$

So, $P(\text{red and red}) = P(\text{red}) \times P(\text{red})$

$$= \frac{7}{20} \times \frac{7}{20}$$

$$= \frac{49}{400}$$

b) $P(\text{not red}) = P(\text{white or blue})$

$$= \frac{13}{20}$$

So, $P(\text{not red, blue}) = P(\text{not red}) \times P(\text{blue})$

$$= \frac{13}{20} \times \frac{4}{20}$$

$$= \frac{13}{20} \times \frac{1}{5}$$

$$= \frac{13}{100}$$

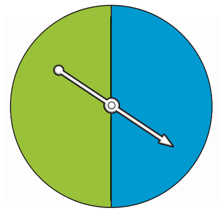
Discuss the ideas

1. In a word problem, what are some words that can be used to suggest the events are independent?
2. In *Example 1*, how can you find the probability of *not* rolling an 8?

Practice

Check

3. A spinner has 2 congruent sectors coloured blue and green. The pointer is spun once, and a coin is tossed.

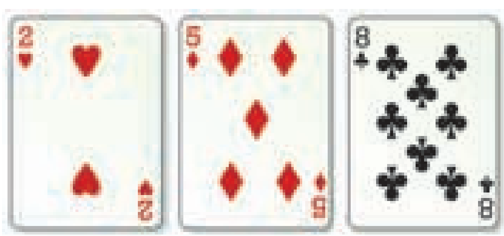


Find the probability of each event:

a) blue and tails

b) blue or green and heads

4. Stanley has two sets of three cards face down on a table. Each set contains: the 2 of hearts, the 5 of diamonds, and the 8 of clubs. He randomly turns over one card from each set.



Find the probability of each event:

a) Both cards are red.

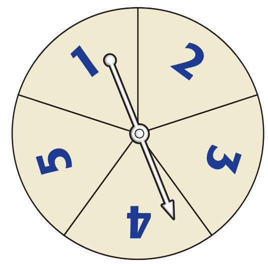
b) The first card is red and the second card is black.

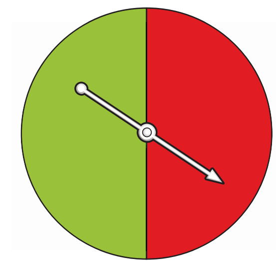
c) Both cards are even numbers.

d) The sum of the numbers is greater than 8.

Which strategy did you use each time?

5. Raoul spins the pointer on each spinner. Find the probability of each event.





a) green and a 2

b) red and an even number

c) green and a prime number

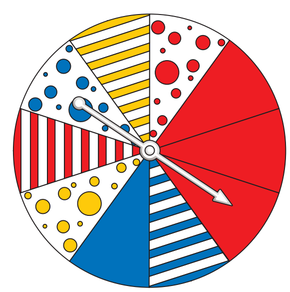
Use a tree diagram or a table to verify your answers.

Apply

6. Find the probability of each event:

a) **i)** The pointer lands on a blue spotted sector, then a solid red sector.

ii) The pointer lands on a red sector, then a spotted sector.



iii) The pointer lands on a striped sector, then a solid blue sector.

iv) The pointer lands on a blue or red sector, then a spotted sector.

b) Use a different strategy to verify your answers in part a.

7. Bart and Bethany play a game. They each roll a regular 6-sided die labelled 1 to 6. Find the probability of each event:

a) Each player rolls a 6.

b) Bart rolls a 6 and Bethany rolls a 2.

c) Bart does not roll a 4 and Bethany rolls an even number.

d) Bart rolls an even number and Bethany rolls an odd number.

e) Bart rolls a number greater than 3 and Bethany rolls a number less than 4.

8. An experiment consists of rolling a die labelled 3 to 8 and picking a card at random from a standard deck of playing cards.

a) What is the probability of each event?

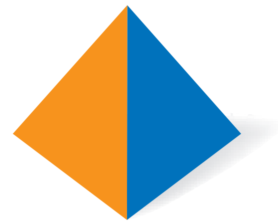
i) rolling a 6 and picking a spade

ii) not rolling a 4 and picking an ace

b) Use a tree diagram to verify your answer to part a, i.

c) What is the probability of picking the ace of spades and rolling a 5? What is the advantage of using the rule instead of a tree diagram?

9. A game at a school carnival involves rolling a regular tetrahedron. Its four faces are coloured red, orange, blue, and green. A player rolls the tetrahedron twice. To win, a player must roll the same colour both times. Marcus has been watching the game. He says he has figured out the probability of a player winning. "The probability of rolling any colour is $\frac{1}{4}$. So, the probability of rolling the same colour again is $\frac{1}{4}$. Since the events are independent, the probability of rolling the same colour both times is $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$."
Do you agree with Marcus?
Justify your answer.
Use a tree diagram to show your thinking.



10. A dresser drawer contains five pairs of socks of these colours: blue, brown, green, white, and black. The socks in each pair are folded together.
Pinto reaches into the drawer and takes a pair of socks without looking.
He wants a black pair.

a) What is the probability that Pinto takes the black pair of socks on his first try?

b) What is the probability that Pinto takes the green pair of socks on his first and second tries?

c) What assumptions do you make?

11. Suppose it is equally likely that a baby be born a boy or a girl.

a) What is the probability that, in a family of 2 children, both children will be boys?

b) Verify your answer to part a using a different method.

12. **Assessment Focus** A bag contains 6 red marbles, 4 blue marbles, and 2 yellow marbles. A student removes 1 marble without looking, records the colour, then returns the marble to the bag. The process is repeated.

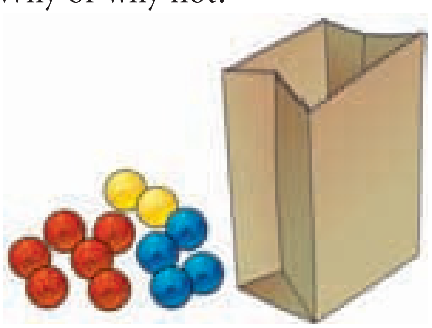
a) What is the probability of each outcome?

i) a red marble, then a yellow marble

ii) 2 blue marbles

iii) not a blue marble, then a yellow marble

b) Suppose the marbles are not returned to the bag. Could you use the rule for two independent events to find each probability in part a? Why or why not?



13. Luke and Salina play the card game "Slam." Each player has 10 cards numbered 1 to 10. Both players turn over one card at the same time. The player whose card has the greater value gets one point. If both cards are the same, a tie is declared and no point is given. After each round, the cards are returned to the pile and all the cards are shuffled.

Find the probability of each event:

a) Luke will get one point when he turns over a 3.

b) Salina will get one point when she turns over a 10.

c) Luke and Salina will tie.

d) Salina will get one point when she turns over a 1.

14. **Take It Further** Neither Andrew nor David like to set the table for dinner. They each toss a coin to decide who will set the table. If both coins show heads, David sets the table. If both coins show tails, Andrew sets the table. If the coins show a head and a tail, both Andrew and David set the table.

What is the probability David will set the table alone 2 days in a row?

Show your work.

15. **Take It Further** A coin is tossed and a die labelled 1 to 6 is rolled.

Write an event that has each probability below.

a) $\frac{1}{2}$ **b)** $\frac{1}{6}$ **c)** $\frac{1}{3}$

Reflect

Which method of finding the probability of 2 independent events do you prefer? Why?

When might the rule not be the best method?

When might the tree diagram or table not be the best method?

Include an example in your explanation.