In Lessons 5.1 and 5.2, you used models to add fractions.
You may not always have suitable models.

You need a strategy you can use to add fractions
without using a model.

Explore

Copy these diagrams.

$$\frac{\square}{\square} + \frac{\square}{\square} = \qquad \frac{\square}{\square} + \frac{\square}{\square} =$$

greatest sum   least sum

Use the digits 1, 2, 4, and 8 to make
the greatest sum and the least sum.
In each case, use each digit once.

Reflect & Share

Share your results with another pair of classmates.
Did you have the same answers?
If not, which is the greatest sum? The least sum?
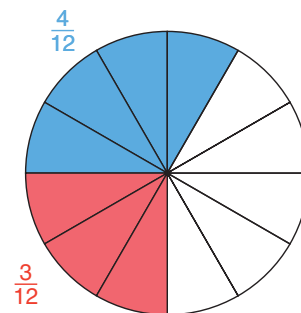What strategies did you use to add?

Connect

We can use equivalent fractions to add $\frac{1}{4} + \frac{1}{3}$.
Use equivalent fractions that have like denominators.
12 is a multiple of 3 and 4.
12 is a **common denominator**.
$$\frac{1}{4} = \frac{3}{12} \quad \text{and} \quad \frac{1}{3} = \frac{4}{12}$$
$$\text{So,} \frac{1}{4} + \frac{1}{3} = \frac{3}{12} + \frac{4}{12}$$
$$= \frac{7}{12}$$

Both fractions are written
with like denominators.

$\frac{4}{12}$

$\frac{3}{12}$

Look at the pattern in the equivalent fractions below.

$$\overset{\times 3}{\frac{1}{4}} = \frac{3}{12} \overset{\times 3}{}$$

$$\overset{\times 4}{\frac{1}{3}} = \frac{4}{12} \overset{\times 4}{}$$

So, to get an equivalent fraction, multiply the numerator and denominator by the same number.

We may also get equivalent fractions by dividing.

For example, $\frac{8}{10}$ can be written: $\frac{8 \div 2}{10 \div 2} = \frac{4}{5}$

$\frac{8}{10}$ and $\frac{4}{5}$ are equivalent fractions.

$\frac{4}{5}$ is in simplest form.

Example

Add: $\frac{4}{9} + \frac{5}{6}$

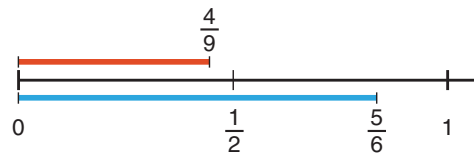Estimate to check the sum is reasonable.

A Solution

$\frac{4}{9} + \frac{5}{6}$

Estimate first.

$\frac{4}{9}$ is about $\frac{1}{2}$.

$\frac{5}{6}$ is close to 1.

So, $\frac{4}{9} + \frac{5}{6}$ is about $1\frac{1}{2}$.

Use equivalent fractions to write the fractions with a common denominator.

List the multiples of 9: 9, **18**, 27, *36*, 45, …

List the multiples of 6: 6, 12, **18**, 24, 30, *36*, 42, …

18 is a multiple of 9 and 6, so 18 is a common denominator.

36 is also in both lists. So, 36 is another possible common denominator.
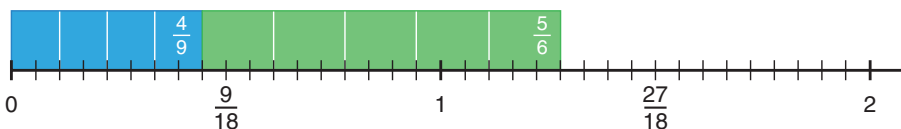
$$\overset{\times 2}{\frac{4}{9}} = \frac{8}{18} \overset{\times 2}{}$$

$$\overset{\times 3}{\frac{5}{6}} = \frac{15}{18} \overset{\times 3}{}$$

$\frac{4}{9} + \frac{5}{6} = \frac{8}{18} + \frac{15}{18}$ Add the numerators.

$\qquad\quad = \frac{23}{18}$

We could have found this sum with fraction strips on a number line.

Since 23 > 18, this is an improper fraction.

To write the fraction as a mixed number:

$\frac{23}{18} = \frac{18}{18} + \frac{5}{18}$

$\quad = 1 + \frac{5}{18}$

$\quad = 1\frac{5}{18}$ This is a mixed number.

The estimate was $1\frac{1}{2}$, so the answer is reasonable.

> **Recall that an improper fraction is a fraction with the numerator greater than the denominator.**

Practice

Write all sums in simplest form.

Write improper fractions as mixed numbers.

1. Find a common denominator for each pair of fractions.

   a) $\frac{1}{2}$ and $\frac{5}{8}$ b) $\frac{1}{8}$ and $\frac{2}{3}$ c) $\frac{2}{3}$ and $\frac{1}{9}$ d) $\frac{3}{5}$ and $\frac{2}{3}$

2. Copy and complete. Replace each $\square$ with a digit to make each equation true.

   a) $\frac{3}{12} = \frac{\square}{4}$ b) $\frac{3}{4} = \frac{6}{\square}$ c) $\frac{3}{6} = \frac{\square}{4}$ d) $\frac{6}{8} = \frac{15}{\square}$

3. Add. Sketch a number line to model each sum.

   a) $\frac{4}{9} + \frac{1}{3}$ b) $\frac{1}{2} + \frac{1}{3}$ c) $\frac{3}{8} + \frac{3}{2}$ d) $\frac{3}{4} + \frac{1}{6}$

4. Estimate, then add.

   a) $\frac{3}{5} + \frac{4}{8}$ b) $\frac{1}{6} + \frac{5}{8}$ c) $\frac{5}{6} + \frac{7}{9}$

   d) $\frac{3}{4} + \frac{4}{7}$ e) $\frac{1}{3} + \frac{2}{5}$ f) $\frac{1}{5} + \frac{5}{6}$

5. One page of a magazine had 2 advertisements.
   One was $\frac{1}{8}$ of the page, the other $\frac{1}{16}$.
   What fraction of the page was covered?
   Show your work.

6. Which sum is greater? Show your thinking.

$\frac{2}{3} + \frac{5}{6}$ or $\frac{3}{4} + \frac{4}{5}$

7. Assessment Focus Three people shared a pie.
Which statement is true? Can both statements be true?
Use pictures to show your thinking.

a) Edna ate $\frac{1}{10}$, Farrah ate $\frac{3}{5}$, and Ferris ate $\frac{1}{2}$.

b) Edna ate $\frac{3}{10}$, Farrah ate $\frac{1}{5}$, and Ferris ate $\frac{1}{2}$.

8. Damara and Baldwin had to shovel snow to clear their driveway.
Damara shovelled about $\frac{3}{10}$ of the driveway.
Baldwin shovelled about $\frac{2}{3}$ of the driveway.
What fraction of the driveway was cleared of snow?

9. Each fraction below is written as the sum of two unit fractions.
Which sums are correct? Why do you think so?

A fraction with numerator 1 is a unit fraction.

a) $\frac{7}{10} = \frac{1}{5} + \frac{1}{2}$ b) $\frac{5}{12} = \frac{1}{3} + \frac{1}{4}$ c) $\frac{5}{6} = \frac{1}{3} + \frac{1}{3}$

d) $\frac{7}{12} = \frac{1}{2} + \frac{1}{6}$ e) $\frac{11}{18} = \frac{1}{2} + \frac{1}{9}$ f) $\frac{2}{15} = \frac{1}{10} + \frac{1}{30}$

10. Take It Further Add.

a) $\frac{3}{8} + \frac{1}{2} + \frac{3}{4}$ b) $\frac{1}{4} + \frac{3}{2} + \frac{2}{5}$ c) $\frac{2}{3} + \frac{5}{6} + \frac{4}{9}$

Reflect

Suppose your friend has forgotten how to add
two fractions with unlike denominators.
What would you do to help?