4.5 Area of a Circle

Focus Develop and use a formula to find the area of a circle.

Explore

You will need one set of fraction circles, masking tape, and a ruler.

➤ Each of you chooses one circle from the set of fraction circles. The circle you choose should have an even number of sectors, and at least 4 sectors.

➤ Each of you cuts 3 strips of masking tape:
- 2 short strips
- 1 strip at least 15 cm long

Use the short strips to fasten the long strip face up on the table.

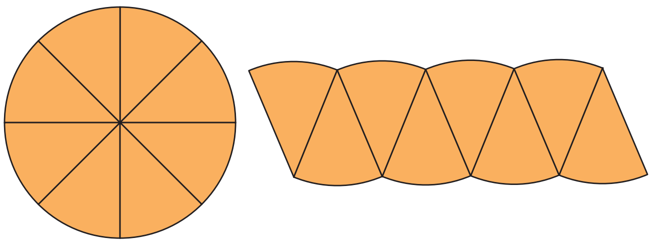
➤ Arrange all your circle sectors on the tape to approximate a parallelogram.
Trace your parallelogram, then use a ruler to make the horizontal sides straight.
Calculate the area of the parallelogram. Estimate the area of the circle.
How does the area of the parallelogram compare to the area of the circle?

Reflect & Share

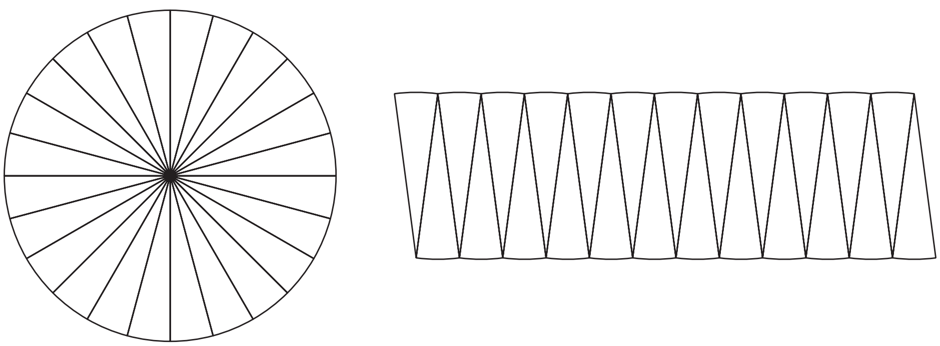
Compare your measure of the area of the circle with the measures of your group members.
Which area do you think is closest to the area of the circle? Why?
How could you improve your estimate for the area?
Which circle measure best represents the height of the parallelogram?
The base? Work together to write a formula for the area of a circle.

Connect

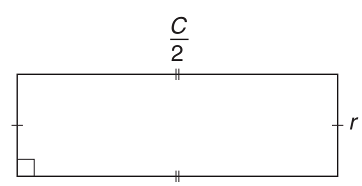
Suppose a circle was cut into 8 congruent sectors.
The 8 sectors were then arranged to approximate a parallelogram.



The more congruent sectors we use, the closer the area of the parallelogram is to the area of the circle.
Here is a circle cut into 24 congruent sectors.
The 24 sectors were then arranged to approximate a parallelogram.



The greater the number of sectors, the more the shape looks like a rectangle.
The sum of the two longer sides of the rectangle is equal to the circumference, C.
So, each longer side, or the base of the rectangle, is one-half the circumference of the circle, or $\frac{C}{2}$.

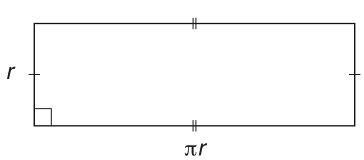


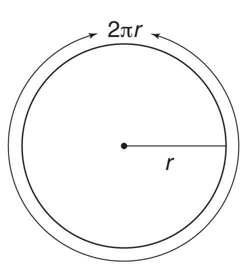
But $C = 2\pi r$

So, the base of the rectangle $= \frac{2\pi r}{2}$

$= \pi r$

Each of the two shorter sides is equal to the radius, r.

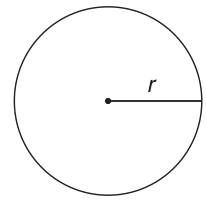




The area of a rectangle is: base $\times$ height
The base is πr. The height is r.
So, the area of the rectangle is: $\pi r \times r = \pi r^2$

When a number or variable is multiplied by itself we write: $7 \times 7 = 7^2$
$r \times r = r^2$

Since the rectangle is made from all sectors of the circle, the rectangle and the circle have the same area.
So, the area, A, of the circle with radius r is $A = \pi r^2$.



$A = \pi r^2$

We can use this formula to find the area of any circle when we know its radius.

Example

The face of a dime has diameter 1.8 cm.

a) Calculate the area.
Give the answer to two decimal places.

b) Estimate to check the answer is reasonable.

A Solution

The diameter of the face of a dime is 1.8 cm.

So, its radius is: $\frac{1.8 \text{ cm}}{2} = 0.9$ cm

a) Use the formula: $A = \pi r^2$
Substitute: $r = 0.9$
$A = \pi \times 0.9^2$
Use a calculator.
$A \doteq 2.544\ 69$
The area of the face of the dime is 2.54 cm^2 to two decimal places.

If your calculator does not have an x^2 key, key in 0.9×0.9 instead.

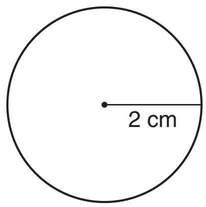
b) Recall that $\pi \doteq 3$.
So, the area of the face of the dime is about $3r^2$.

$$r \doteq 1$$
$$\text{So, } r^2 = 1$$
$$\text{and } 3r^2 = 3 \times 1$$
$$= 3$$

The area of the face of the dime is approximately 3 cm^2.
Since the calculated area, 2.54 cm^2, is close to 3 cm^2, the answer is reasonable.

Practice

1. Calculate the area of each circle.
Estimate to check your answers are reasonable.

a)


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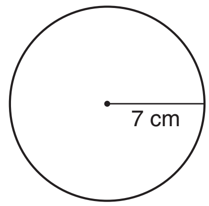

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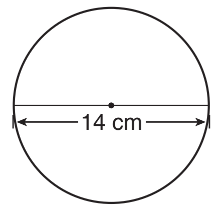

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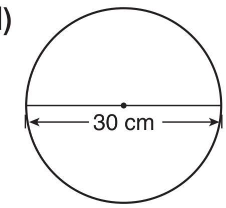
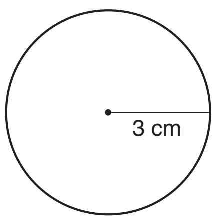


2. Calculate the area of each circle. Give your answers to two decimal places.
Estimate to check your answers are reasonable.

a)


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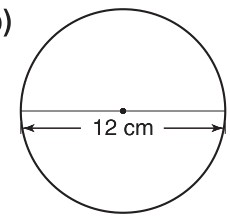

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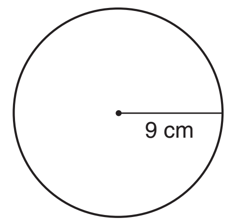

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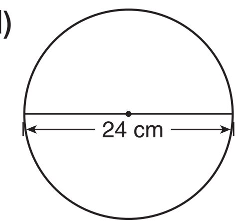
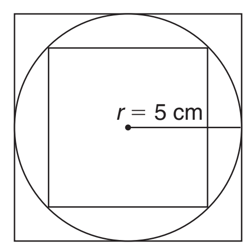


3. Use the results of questions 1 and 2. What happens to the area in each case?

a) You double the radius of a circle.
b) You triple the radius of a circle.
c) You quadruple the radius of a circle.

Justify your answers.

4. **Assessment Focus** Use 1-cm grid paper.
Draw a circle with radius 5 cm.
Draw a square outside the circle that just encloses the circle.
Draw a square inside the circle
so that its vertices lie on the circle.
Measure the sides of the squares.



a) How can you use the areas of the two squares to estimate the area of the circle?
b) Check your estimate in part a by calculating the area of the circle.
c) Repeat the activity for circles with different radii.
Record your results. Show your work.

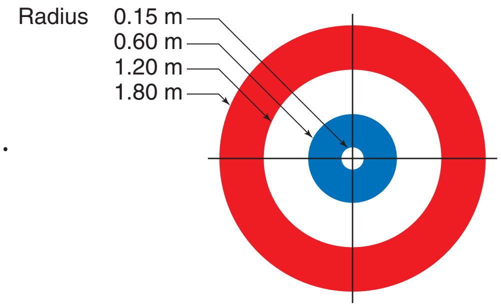
5. In the biathlon, athletes shoot at targets. Find the area of each target.

a) The target for the athlete who is standing is a circle with diameter 11.5 cm.
b) The target for the athlete who is lying down is a circle with diameter 4.5 cm.

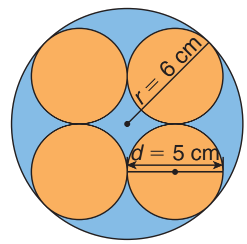
Give the answers to the nearest square centimetre.

6. In curling, the target area is a bull's eye with 4 concentric circles.
   a) Calculate the area of the smallest circle.
   b) When a smaller circle overlaps a larger circle, a ring is formed.
      Calculate the area of each ring on the target area.
   Give your answers to 4 decimal places.

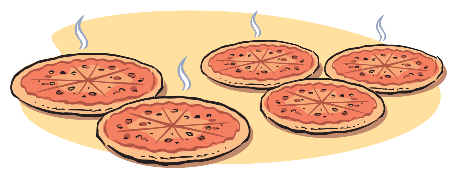
Concentric circles have the same centre.



7. **Take It Further**
   A circle with radius 6 cm contains 4 small circles.
   Each small circle has diameter 5 cm.
   Each small circle touches two other small circles and the large circle.
   a) Find the area of the large circle.
   b) Find the area of one small circle.
   c) Find the area of the region that is shaded yellow.



8. **Take It Further** A large pizza has diameter 35 cm.
   Two large pizzas cost $19.99.
   A medium pizza has diameter 30 cm.
   Three medium pizzas cost $24.99.
   Which is the better deal: 2 large pizzas or 3 medium pizzas?
   Justify your answer.



Agriculture: Crop Circles

In Red Deer, Alberta, on September 17, 2001, a crop circle formation was discovered that contained 7 circles. The circle shown has diameter about 10 m. This circle destroyed some wheat crop. What area of wheat crop was lost in this crop circle?

You have learned two formulas for measurements of a circle.
How do you remember which formula to use for the area of a circle?