6.3 Solving Equations Involving Fractions

Focus Solve an equation that involves a fraction.

Which number could you multiply $\frac{3}{7}$ by to get the product 3?

Which number could you multiply $\frac{5}{6}$ by to get the product 5?
How did you find out?

Investigate

Work with a partner.

A fair comes to Behchoko, NWT.
Nicole has some tickets for the midway.
She shares the tickets with 2 friends so that they have 9 tickets each.
How many tickets did Nicole begin with?

Let t represent the number of tickets Nicole began with.
Write an equation you can use to solve for t.
Use any strategy to solve the problem.

Reflect & Share

Compare the equation you wrote with that of another pair of classmates.

If the equations are different, are both equations correct?
How do you know?

If you did not write an equation that involved division, work together to do so now.
What strategy would you use to solve this equation?
How could you check your answer?

Equations that involve fractions cannot be easily modelled with algebra tiles or balance scales.
We can write and solve these equations using algebra.

Example 1

Grandpa has enough gift certificates to give the same number to each of his 4 grandchildren.
After Grandpa gives them the gift certificates, each grandchild has 5 gift certificates.
How many gift certificates does Grandpa have?

a) Write an equation to represent this problem.
b) Solve the equation.
c) Verify the solution.

A Solution

a) Let n represent the number of gift certificates Grandpa has.
He shares them equally among 4 grandchildren.
Each grandchild will receive $\frac{n}{4}$ gift certificates.
Each grandchild will then have 5 gift certificates.
One possible equation is: $\frac{n}{4} = 5$

b) Solve the equation using algebra.

$\frac{n}{4} = 5$ To isolate the variable, multiply each side by 4.

$\frac{n}{4} \times 4 = 5 \times 4$

$n = 20$

Grandpa has 20 gift certificates.

c) To verify the solution,
substitute $n = 20$ into $\frac{n}{4} = 5$.

Left side $= \frac{n}{4}$ Right side $= 5$

$= \frac{20}{4}$

$= 5$

Since the left side equals the right side, $n = 20$ is correct.
Grandpa has 20 gift certificates.

Example 2

The school's student council sold T-shirts for charity.
The council bought the T-shirts in boxes of 40.
The student council added $6 to the cost of each T-shirt.
Each T-shirt sold for $26.
What did the student council pay for 1 box of T-shirts?

a) Write an equation to represent this problem.
Solve the equation.

b) Verify the solution.

A Solution

a) Let c dollars represent what the student council paid for 1 box of T-shirts.
The cost of each T-shirt the student council bought was: $\frac{c}{40}$
$6 was added to the cost of each T-shirt: $\frac{c}{40} + 6$
Each T-shirt sold for $26.
So, an equation is: $\frac{c}{40} + 6 = 26$

Solve the equation using algebra.

$\frac{c}{40} + 6 = 26$ To isolate the variable term, subtract 6 from each side.

$\frac{c}{40} + 6 - 6 = 26 - 6$

$\frac{c}{40} = 20$ To isolate the variable, multiply each side by 40.

$\frac{c}{40} \times 40 = 20 \times 40$

$c = 800$

The student council paid $800 for 1 box of T-shirts.

b) To verify the solution,
substitute $c = 800$ into $\frac{c}{40} + 6 = 26$.

Left side $= \frac{c}{40} + 6$ Right side = 26

$= \frac{800}{40} + 6$

$= 20 + 6$

$= 26$

Since the left side equals the right side, $c = 800$ is correct.

Discuss the ideas

1. Why would it be difficult to model and solve equations like those in *Examples 1* and *2* with algebra tiles or balance scales?
2. In *Example 2*, why did we subtract 6 from each side before we multiplied by 40?

Practice

Check

3. Solve each equation.
Verify the solution.

a) $\frac{t}{5} = 6$ **b)** $\frac{a}{7} = 8$

c) $\frac{b}{6} = 3$ **d)** $\frac{c}{3} = 9$

4. Solve each equation.
Verify the solution.

a) $\frac{d}{-4} = 5$ **b)** $\frac{f}{8} = -5$

c) $\frac{k}{9} = -4$ **d)** $\frac{m}{-5} = -7$

5. One-quarter of the golf balls in the bag are yellow.
There are 8 yellow golf balls.
How many golf balls are in the bag?

a) Write an equation you can use to solve the problem.
b) Solve the equation.
c) Verify the solution.

6. For each sentence, write an equation.
Solve the equation to find the number.

a) A number divided by 6 is 9.
b) A number divided by -4 is -3.
c) A number divided by -5 is 7.

7. Solve each equation.
Verify the solution.

a) $\frac{n}{4} + 3 = 10$ **b)** $\frac{m}{3} - 2 = 9$

c) $13 + \frac{x}{2} = 25$ **d)** $-9 + \frac{s}{2} = 2$

Apply

8. Solve each equation.
Verify the solution.

a) $\frac{p}{-3} + 9 = 3$ **b)** $\frac{t}{-6} + 12 = 18$

c) $-24 + \frac{w}{5} = -29$ **d)** $-17 + \frac{e}{-7} = -8$

9. For each sentence, write an equation.
Solve the equation to find the number.

a) Add 1 to a number divided by -3 and the sum is 6.
b) Subtract a number divided by 9 from 3 and the difference is 0.
c) Add 4 to a number divided by -2 and the sum is -3.

10. One-half of the team's supply of baseballs was taken from the dressing room to the dugout. During the game, 11 baseballs were caught by fans. At the end of the game, there were 12 baseballs left in the dugout. What was the team's original supply of baseballs?

a) Write an equation you can use to solve the problem.
b) Solve the equation.
c) Verify the solution.

11. **Assessment Focus** Five students in Mrs. Lamert's tutorial class after school are solving equations. She brought a bag of treats. Mrs. Lamert explained that if the 5 students shared the bag of treats equally, then gave one treat each to the teacher, each student would still have 9 treats. How many treats were in the bag? Here is the equation Jerry suggested: $\frac{n}{5} - 1 = 9$

a) Is Jerry's equation correct? Explain why or why not.

b) If your answer to part a is yes, solve the equation using algebra. If your answer to part a is no, correct the equation, then solve the equation using algebra.

c) Verify the solution.

12. One-third of the Grade 8 students went to the track-and-field meet. Five track coaches went too. There were 41 people on the bus, not including the driver. How many students are in Grade 8?

a) Write an equation you can use to solve the problem.

b) Solve the equation.

c) Verify the solution.

13. Check each student's work below. Rewrite a correct and complete algebraic solution where necessary.

a) Student A:

$$\frac{h}{-9} = 4$$
$$\frac{h}{-9} \times (-9) = 4 \times (-9)$$
$$\frac{-9h}{-9} = -36$$
$$h = -36$$

b) Student B:

$$\frac{t}{8} - 2 = 4$$
$$8 \times \frac{t}{8} - 2 = 4 \times 8$$
$$\frac{8t}{8} - 2 = 32$$
$$t - 2 = 32$$
$$t - 2 + 2 = 32 + 2$$
$$t = 34$$

c) Student C:

$$\frac{r}{-4} + 3 = 13$$
$$\frac{r}{-4} + 3 - 3 = 13 - 3$$
$$\frac{r}{-4} = 10$$
$$4 \times \frac{r}{-4} = 10 \times 4$$
$$r = 40$$

14. **Take It Further** Jonah used the equation $3 + \frac{n}{7} = 18$ to solve a word problem.

a) What might the word problem be?

b) Solve the problem.

c) Verify the solution.

Reflect

How did your knowledge of multiplying fractions help you in this lesson? Include examples in your explanation.