

# 3.6

## Dividing Fractions

### Focus

Develop algorithms to divide fractions.

You have used grouping to divide a whole number by a fraction:  $4 \div \frac{2}{3} = 6$

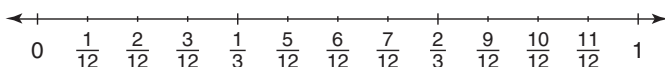
You have used sharing to divide a fraction by a whole number:  $\frac{2}{3} \div 4 = \frac{1}{6}$

You will now investigate dividing a fraction by a fraction.

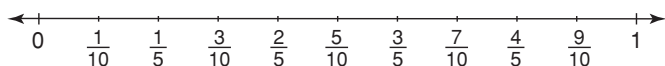
### Investigate

Work with a partner.

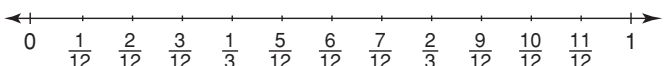
Use this number line. Find:  $\frac{2}{3} \div \frac{1}{4}$



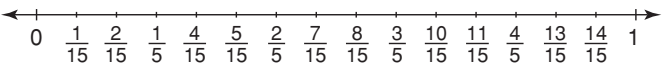
Use this number line. Find:  $\frac{3}{5} \div \frac{1}{2}$



Use this number line. Find:  $\frac{1}{3} \div \frac{1}{4}$



Use this number line. Find:  $\frac{4}{5} \div \frac{2}{3}$



Look at the quotients.

How do the numbers in the numerators and denominators relate to the quotients?

Try to find a strategy to calculate the quotient without using a number line.

Use a different division problem to check your strategy.

### Reflect & Share

Compare your strategy with that of another pair of classmates.

Does your strategy work with their problem? Explain.

Does their strategy work with your problem? Explain.

## Connect

Here are two ways to divide fractions.

- Use common denominators.

To divide:  $\frac{4}{5} \div \frac{1}{10}$

Write each fraction with a common denominator.

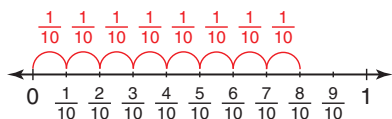
Since 5 is a factor of 10, 10 is a common denominator.

$$\frac{4}{5} = \frac{8}{10}$$

(Diagram showing  $\frac{4}{5}$  on the left and  $\frac{8}{10}$  on the right, with an equals sign between them. A red arrow points from  $\frac{4}{5}$  to  $\frac{8}{10}$  with the label  $\times 2$ . Another red arrow points from  $\frac{8}{10}$  back to  $\frac{4}{5}$  with the label  $\times 2$ .)

So,  $\frac{4}{5} \div \frac{1}{10} = \frac{8}{10} \div \frac{1}{10}$

This means: How many 1 tenths are in 8 tenths?



From the number line, this is the same as  $8 \div 1 = 8$ .

So,  $\frac{4}{5} \div \frac{1}{10} = 8$

**When the denominators are the same, divide the numerators.**

- Use multiplication.

Every whole number can be written as a fraction with denominator 1.

Here are some division equations from Lesson 3.5 and their related multiplication equations.

Each multiplication equation uses the reciprocal of the divisor.

This pattern is always true.

Division Equation	Related Multiplication Equation
$5 \div \frac{3}{5} = \frac{25}{3}$ or $\frac{5}{1} \div \frac{3}{5} = \frac{25}{3}$	$\frac{5}{1} \times \frac{5}{3} = \frac{25}{3}$
$4 \div \frac{3}{4} = \frac{16}{3}$ or $\frac{4}{1} \div \frac{3}{4} = \frac{16}{3}$	$\frac{4}{1} \times \frac{4}{3} = \frac{16}{3}$
$6 \div \frac{5}{3} = \frac{18}{5}$ or $\frac{6}{1} \div \frac{5}{3} = \frac{18}{5}$	$\frac{6}{1} \times \frac{3}{5} = \frac{18}{5}$
$\frac{1}{2} \div 3 = \frac{1}{6}$ or $\frac{1}{2} \div \frac{3}{1} = \frac{1}{6}$	$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$
$\frac{3}{4} \div 2 = \frac{3}{8}$ or $\frac{3}{4} \div \frac{2}{1} = \frac{3}{8}$	$\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$
$\frac{4}{8} \div 4 = \frac{1}{8}$ or $\frac{4}{8} \div \frac{4}{1} = \frac{1}{8}$	$\frac{4}{8} \times \frac{1}{4} = \frac{1}{8}$

We can use the same patterns to divide two fractions.

For example, to find  $\frac{3}{5} \div \frac{1}{4}$ , write the reciprocal of the divisor, then multiply.

$$\begin{aligned} \frac{3}{5} \div \frac{1}{4} &= \frac{3}{5} \times \frac{4}{1} \\ &= \frac{12}{5}, \text{ or } 2\frac{2}{5} \end{aligned}$$

### Example 1

Divide. Estimate to check each quotient is reasonable.

$$\frac{3}{4} \div \frac{5}{6}$$

#### ▶ A Solution

$$\frac{3}{4} \div \frac{5}{6}$$

Use multiplication.

$$\frac{3}{4} \div \frac{5}{6} \text{ can be written as}$$

$$\begin{aligned} \frac{3}{4} \times \frac{6}{5} &= \frac{3 \times \cancel{6}^3}{\cancel{4}_2 \times 5} \text{ Simplify. Divide by common factor 2.} \\ &= \frac{9}{10} \end{aligned}$$

Estimate to check.

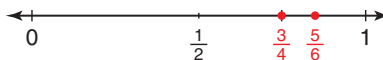
$\frac{3}{4}$  is about 1.  $\frac{5}{6}$  is about 1.

So,  $\frac{3}{4} \div \frac{5}{6}$  is about  $1 \div 1 = 1$ .

Since  $\frac{5}{6}$  is greater than  $\frac{3}{4}$ ,

$\frac{3}{4} \div \frac{5}{6}$  is less than 1.

Since  $\frac{9}{10}$  is close to 1, and less than 1, the quotient is reasonable.



Dividing by  $\frac{5}{6}$  is the same as multiplying by  $\frac{6}{5}$ .

### Example 2

Divide:  $\frac{7}{8} \div \frac{1}{4}$

#### ▶ A Solution

$$\frac{7}{8} \div \frac{1}{4}$$

Use common denominators.

Since 8 is a multiple of 4, 8 is a common denominator.

Multiply the numerator and denominator by 2:  $\frac{1}{4} = \frac{2}{8}$

$$\begin{aligned} \frac{7}{8} \div \frac{1}{4} &= \frac{7}{8} \div \frac{2}{8} \\ &= 7 \div 2 \\ &= \frac{7}{2}, \text{ or } 3\frac{1}{2} \end{aligned}$$

Since the denominators are the same, divide the numerators.

Estimate to check.  $\frac{7}{8}$  is close to 1, but less than 1.

Since there are 4 quarters in one whole,  $\frac{7}{8} \div \frac{1}{4}$  is close to 4, but less than 4.

Since  $3\frac{1}{2}$  is close to 4, and less than 4, the quotient is reasonable.

### Example 3

Use a number line to illustrate the quotient.

$$\frac{3}{5} \div \frac{1}{4}$$

#### A Solution

$$\frac{3}{5} \div \frac{1}{4}$$

Use common denominators.

Write each fraction with a common denominator.

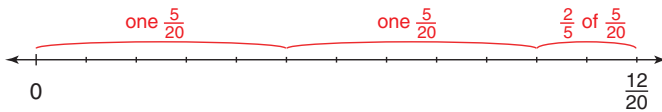
Since 5 and 4 have no common factors, a common denominator is  $5 \times 4 = 20$ .

$$\frac{3}{5} = \frac{12}{20}$$

$$\frac{1}{4} = \frac{5}{20}$$

$$\frac{3}{5} \div \frac{1}{4} = \frac{12}{20} \div \frac{5}{20}$$

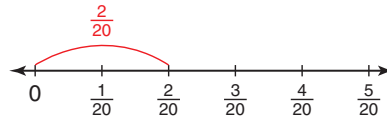
This means: How many 5 twentieths are in 12 twentieths?



From the number line, there are 2 groups of 5 twentieths with remainder 2 twentieths. Write the remainder as a fraction of  $\frac{5}{20}$ .

2 twentieths is  $\frac{2}{5}$  of 5 twentieths.

$$\text{So, } \frac{3}{5} \div \frac{1}{4} = 2\frac{2}{5}$$



### Discuss the ideas

1. Without dividing, how do you know if the quotient of  $\frac{5}{6} \div \frac{2}{3}$  is less than or greater than 1?
2. Why is it important to estimate to check the quotient?
3. What is a strategy for dividing two fractions?

## Practice

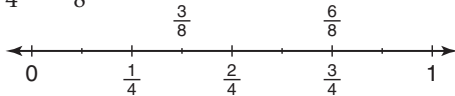
### Check

4. Write the reciprocal of each fraction.

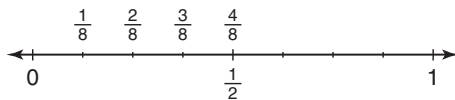
a)  $\frac{5}{9}$       b)  $\frac{3}{7}$       c)  $\frac{7}{8}$       d)  $\frac{14}{15}$

5. Use a copy of each number line to illustrate each quotient.

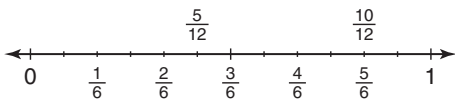
a)  $\frac{3}{4} \div \frac{3}{8}$



b)  $\frac{1}{2} \div \frac{1}{8}$



c)  $\frac{5}{6} \div \frac{5}{12}$



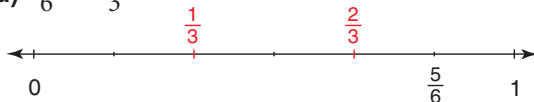
6. Divide:  $\frac{3}{5} \div \frac{9}{10}$

- What is the reciprocal of  $\frac{9}{10}$ ?
- Use multiplication. Simplify first.
- Estimate the quotient.
- Is the quotient reasonable?  
How do you know?

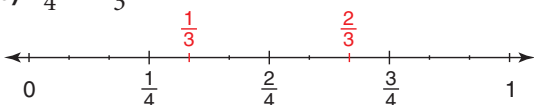
### Apply

7. Use a copy of each number line to illustrate each quotient.

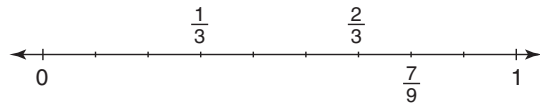
a)  $\frac{5}{6} \div \frac{1}{3}$



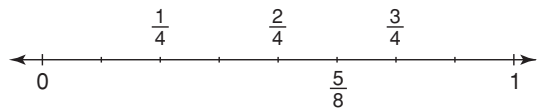
b)  $\frac{3}{4} \div \frac{1}{3}$



c)  $\frac{7}{9} \div \frac{1}{3}$



d)  $\frac{5}{8} \div \frac{1}{4}$



8. Find each quotient.

a)  $\frac{7}{10} \div \frac{3}{10}$       b)  $\frac{5}{9} \div \frac{2}{9}$

c)  $\frac{3}{5} \div \frac{2}{5}$       d)  $\frac{4}{5} \div \frac{2}{5}$

9. Use multiplication to find each quotient.

a)  $\frac{8}{5} \div \frac{3}{4}$       b)  $\frac{9}{10} \div \frac{5}{3}$

c)  $\frac{7}{2} \div \frac{4}{3}$       d)  $\frac{1}{2} \div \frac{7}{6}$

10. Use common denominators to find each quotient.

a)  $\frac{7}{12} \div \frac{1}{4}$       b)  $\frac{3}{5} \div \frac{11}{10}$

c)  $\frac{5}{2} \div \frac{1}{3}$       d)  $\frac{5}{6} \div \frac{9}{8}$

11. Divide. Estimate to check each quotient is reasonable.

a)  $\frac{5}{3} \div \frac{3}{5}$       b)  $\frac{4}{9} \div \frac{4}{9}$       c)  $\frac{1}{6} \div \frac{5}{2}$

12. Suppose you have  $\frac{11}{12}$  of a cake.

How many servings can you make of each size?

a)  $\frac{1}{4}$  of the cake      b)  $\frac{1}{3}$  of the cake

c)  $\frac{1}{6}$  of the cake      d)  $\frac{1}{2}$  of the cake

- 13. a)** Find each quotient.
- i)  $\frac{3}{4} \div \frac{5}{8}$       ii)  $\frac{5}{8} \div \frac{3}{4}$   
 iii)  $\frac{7}{12} \div \frac{2}{5}$       iv)  $\frac{2}{5} \div \frac{7}{12}$   
 v)  $\frac{5}{3} \div \frac{4}{5}$       vi)  $\frac{4}{5} \div \frac{5}{3}$
- b)** In part a, what patterns do you see in the division statements and their quotients? Write two more pairs of division statements that follow the same pattern.
- 14.** As a busboy in a restaurant, Amiel takes  $\frac{1}{12}$  h to clear and reset a table. How many tables can Amiel clear in  $\frac{2}{3}$  h? Estimate to check the solution is reasonable.
- 15.** Divide. Estimate to check each quotient is reasonable.
- a)  $\frac{27}{28} \div \frac{9}{14}$       b)  $\frac{15}{22} \div \frac{3}{11}$   
 c)  $\frac{32}{51} \div \frac{8}{17}$       d)  $\frac{57}{69} \div \frac{19}{115}$
- 16.** To conduct a science experiment, each pair of students requires  $\frac{1}{16}$  cup of vinegar. The science teacher has  $\frac{3}{4}$  cup of vinegar. How many pairs of students can conduct the experiment?

### 17. Assessment Focus

- a)** Copy the boxes below. Write the digits 2, 3, 4, and 5 in the boxes to make as many different division statements as you can.
- $$\frac{\square}{\square} \div \frac{\square}{\square}$$
- b)** Which division statement in part a has the greatest quotient? The least quotient? How do you know? Show your work.
- 18.** Tahoe used the expression  $\frac{7}{8} \div \frac{1}{4}$  to solve a word problem. What might the word problem be? Solve the problem. Estimate to check the solution is reasonable.
- 19. Take It Further** Copy each division equation. Replace each  $\square$  with a fraction to make each equation true. Explain the strategy you used.
- a)  $\frac{2}{3} \div \square = \frac{8}{9}$       b)  $\frac{3}{11} \div \square = \frac{12}{55}$   
 c)  $\frac{1}{4} \div \square = \frac{9}{20}$       d)  $\frac{4}{5} \div \square = \frac{28}{45}$
- 20. Take It Further** Write as many division statements as you can that have a quotient between  $\frac{1}{2}$  and 1. Explain the strategy you used.

## Reflect

Explain how your knowledge of common denominators can help you divide two fractions. Include an example in your explanation.