

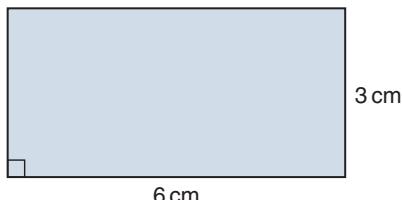
4.4

Surface Area of a Right Triangular Prism

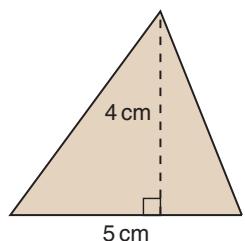
Focus

Find the surface area of a right triangular prism.

What is the area of this rectangle?



What is the area of this triangle?



You will use these measures to find the surface area of a right triangular prism.

Investigate

Work with a partner.

Your teacher will give you a right triangular prism.

- Find the surface area of the triangular prism.
- Describe your strategy for finding the surface area of the prism.
- Do you think your strategy would work for any prism?
Trade prisms with a pair of classmates who used a different prism. Use your strategy to find its surface area.



Reflect & Share

Compare your strategy and results with the same pair of classmates.
Did you use the same strategy?
If not, do both strategies work? Explain.

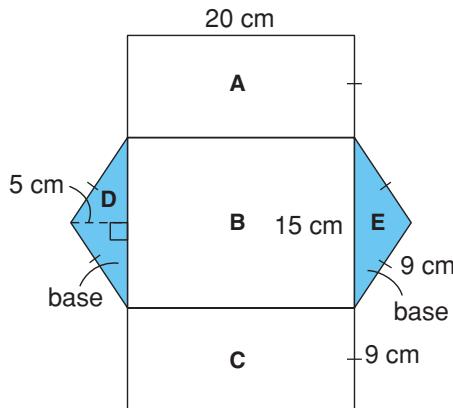
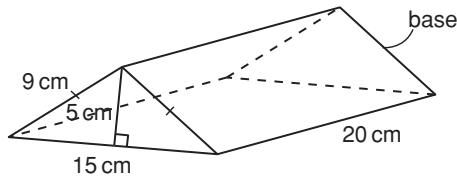
Connect

A right triangular prism has 5 faces: 3 rectangular faces and 2 triangular bases.

The two triangular bases of the prism are congruent.

Here is a triangular prism and its net.

The measurements are given to the nearest centimetre.



The surface area of a triangular prism is the sum of the areas of its faces.

Rectangle A has area: $20 \times 9 = 180$

Rectangle B has area: $20 \times 15 = 300$

Rectangle C has area: $20 \times 9 = 180$

Triangle D has area: $\frac{1}{2} \times 15 \times 5 = \frac{75}{2} = 37.5$

Triangle E has area: $\frac{1}{2} \times 15 \times 5 = \frac{75}{2} = 37.5$

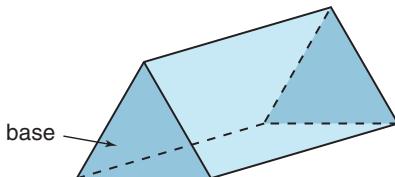
The area measurements are approximate because the measurements are given to the nearest centimetre.

$$\begin{aligned}\text{Surface area} &= \text{area of Rectangle A} + \text{area of Rectangle B} + \text{area of Rectangle C} \\ &\quad + \text{area of Triangle D} + \text{area of Triangle E} \\ &= 180 + 300 + 180 + 37.5 + 37.5 \\ &= 735\end{aligned}$$

The surface area of the prism is 735 cm^2 .

Since the bases of a triangular prism are congruent, we can say:

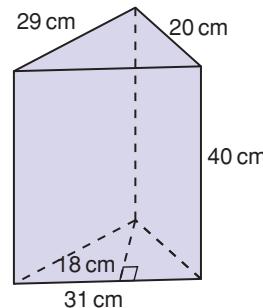
$$\begin{aligned}\text{Surface area} &= \text{sum of the areas of the 3 rectangular faces} \\ &\quad + 2 \times \text{area of one triangular base}\end{aligned}$$



Example 1

Find the surface area of this prism.

Each dimension is given to the nearest centimetre.



A Solution

Draw a net. Label its dimensions.

The prism is 40 cm tall, so each rectangle has height 40 cm.

The width of each rectangle is a side length of the triangular base.

One rectangle has area: $40 \times 20 = 800$

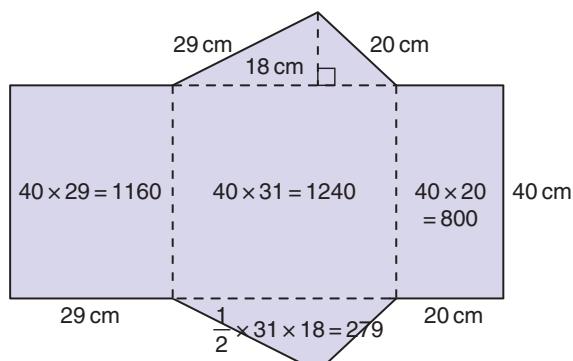
Another rectangle has area: $40 \times 31 = 1240$

The third rectangle has area: $40 \times 29 = 1160$

The triangular base has area: $\frac{1}{2} \times 31 \times 18 = 279$

$$\begin{aligned}\text{The surface area} &= 800 + 1240 + 1160 + 2 \times 279 \\ &= 3758\end{aligned}$$

The surface area of the prism is 3758 cm^2 .

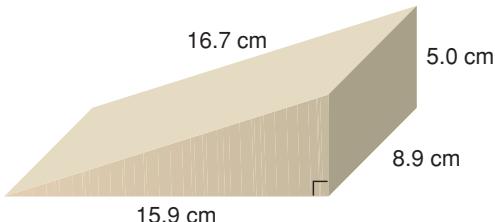


Instead of drawing a net, we can visualize each face as we calculate its area.

Example 2

A wooden doorstop is a triangular prism. It is to be painted. The bottom rectangular face is covered with rubber and will not be painted.

Find the total surface area to be painted.



Each dimension is given to one decimal place.

A Solution

There are 4 faces to be painted: 2 triangular bases, the slanted rectangular face, and the vertical rectangular face.

The area of each triangular base is: $\frac{1}{2} \times 15.9 \times 5.0 = 39.75$

The slanted rectangular face has area: $8.9 \times 16.7 = 148.63$

The vertical rectangular face has area: $5.0 \times 8.9 = 44.5$

Total area to be painted is: $(2 \times 39.75) + 148.63 + 44.5 = 272.63$

The surface area to be painted is 272.6 cm^2 to one decimal place.

Discuss the ideas

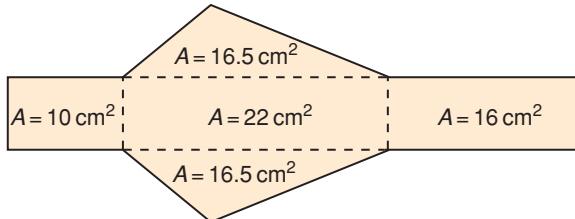
- Explain how the net of a triangular prism can help you find the surface area of the prism.
- What do you know about the rectangular faces in an equilateral triangular prism?
How would you find the surface area of the prism?
- What do you know about the rectangular faces in an isosceles triangular prism?
How would you find the surface area of the prism?

Practice

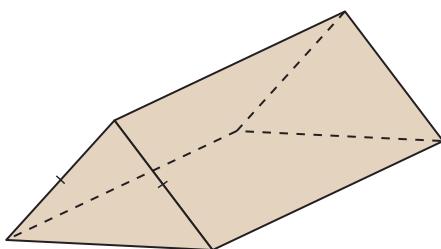
Check

Use a calculator when you need to.

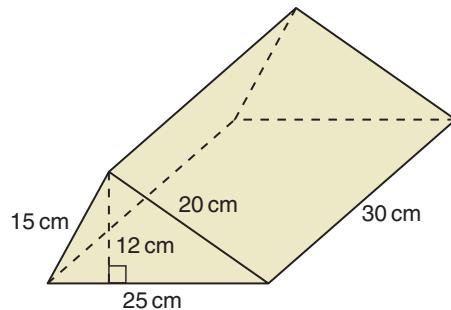
- 4.** Here is the net of a right triangular prism. The area of each face is given. What is the surface area of the prism? How did you find out?



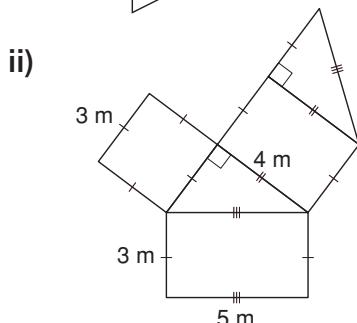
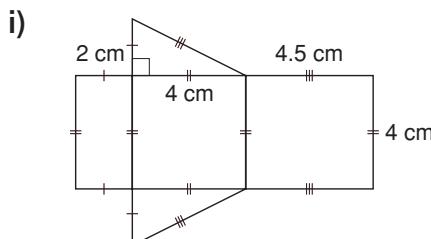
- 5.** Here is a right isosceles triangular prism. Which faces are congruent and share the same area? How do you know?



- 6.** Sketch a net of this triangular prism. What is its surface area?



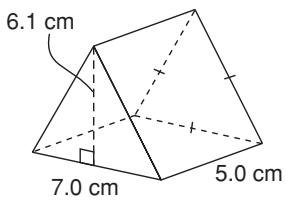
- 7. a)** Calculate the area of each net.



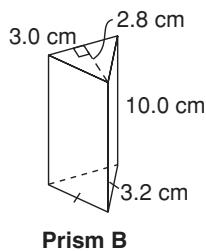
- b)** How does the area of each net compare to the surface area of the prism formed by the net?

Apply

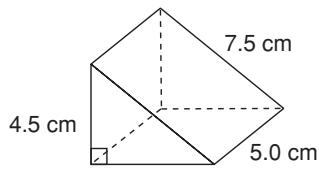
- 8.** Calculate the surface area of each prism. Order the prisms from greatest to least surface area. Show your work.



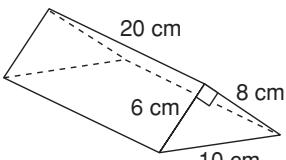
Prism A



Prism B



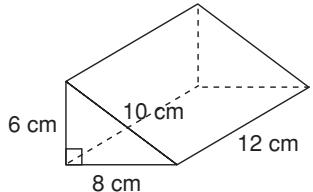
Prism C



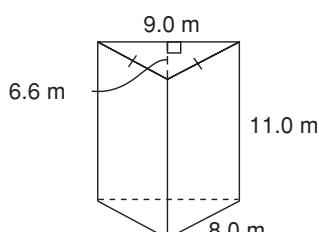
Prism D

- 9.** Find the surface area of each triangular prism.

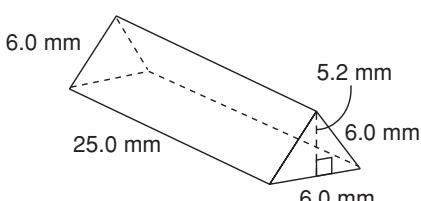
a)



b)



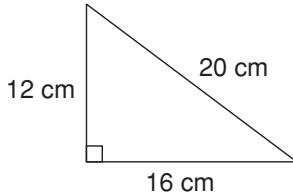
c)



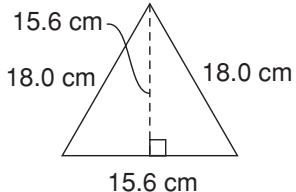
- 10.** The 3 rectangular faces of a triangular prism have areas 30 cm^2 , 40 cm^2 , and 50 cm^2 . The 2 triangular bases have a combined area of 12 cm^2 . What are the dimensions of the triangular prism? Explain your thinking using diagrams, numbers, and words.

- 11.** Suppose you want to construct a right triangular prism 15 cm long with the greatest surface area. Which of these triangles should you choose for its base? Explain your choice.

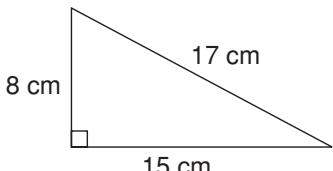
a)



b)

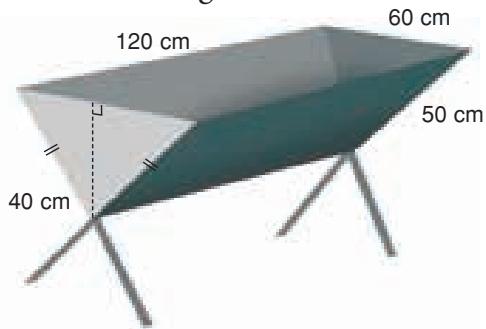


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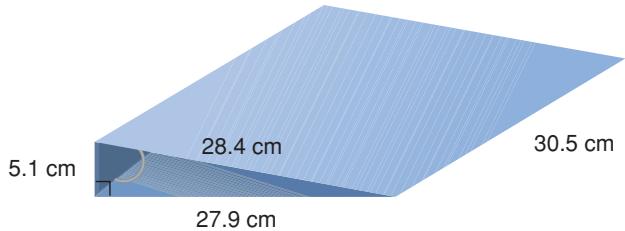


- 12. Assessment Focus** A student said, “If you double all the dimensions of a triangular prism, you will double its surface area.” Is the student correct? Use words, numbers, and diagrams to explain your answer.

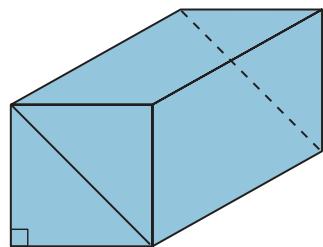
- 13.** How much metal is needed to build this water trough?



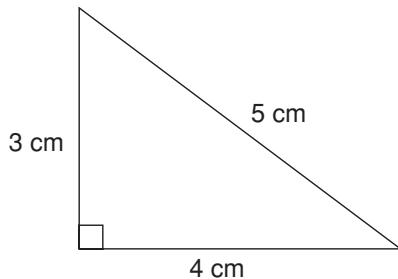
- 14.** Daniel wants to cover the outside of an empty 3-ring binder with plastic. Each dimension of the binder has been written to one decimal place. How much plastic is needed to cover the outside of the binder? What assumptions do you make?



- 15.** A rectangular prism is cut as shown to form two congruent triangular prisms. Is the surface area of one triangular prism one-half the surface area of the rectangular prism? Justify your answer.

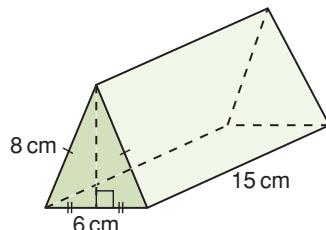


- 16. Take It Further** This triangle is one base of a right triangular prism. What should the length of the prism be so its surface area is between 100 cm^2 and 150 cm^2 ? Show your work.



- 17. Take It Further**

- Use the Pythagorean Theorem. Find the height of a triangular base of this prism.
- What is the surface area of the prism? Give your answer to the nearest square centimetre.



Reflect

How is the strategy for finding the surface area of a triangular prism similar to the strategy for finding the surface area of a rectangular prism? How is it different?