8.4 Constructing Angle Bisectors

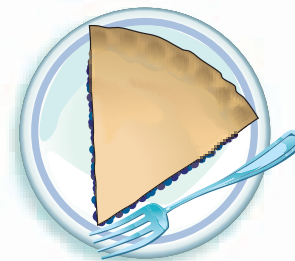You will investigate ways to divide an angle into 2 equal parts.

Explore

Your teacher will give you a large copy of this picture.
You may need rulers, protractors, tracing paper,
plain paper, and Miras.
Use any methods or tools.
George wants to share this slice of pie equally
with a friend.
Show how he could divide the slice of pie into
2 equal parts.

Reflect & Share

Compare your results and methods with those of
another pair of classmates.
How could you use your classmates' methods to divide
the slice of pie in half?

Connect

When you divide an angle into two equal parts, you *bisect* the angle.

Here are 3 strategies to draw the bisector of a given angle.
➤ Use paper folding.
   Fold the paper so that XY lies along ZY.
   Crease along the fold line.
   Open the paper.
   The fold line is the bisector of ∠XYZ.

➤ Use a Mira. Place the Mira so that the reflection
   of one arm of the angle lies along the other arm.
   Draw a line segment along the edge of the Mira.
   This line segment is the bisector of the angle.

➤ Use a plastic right triangle.
  Place the triangle with one angle at B and one edge along BC.
  Draw a line segment.
  Place the triangle with the same angle at B and the same edge along AB.
  Draw a line segment.
 Label M where the line segments you drew intersect. BM is the bisector of ∠ABC.

We can use the properties of a rhombus to construct the bisector of an angle.
Think of the angle as one angle of a rhombus.

Example

Draw obtuse ∠B of measure 126°.
Use a ruler and a compass to bisect the angle.
Measure the angles to check.

A Solution
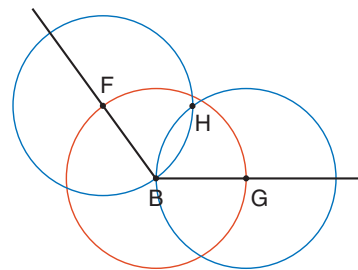
Use a ruler and protractor to draw ∠B = 126°.
Use ∠B as one angle of a rhombus.
With compass point on B, draw a circle that
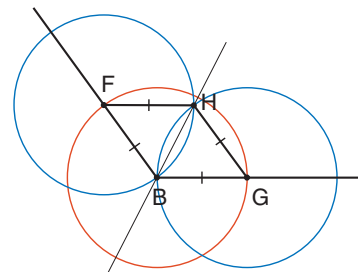intersects one arm at F and the other arm at G.
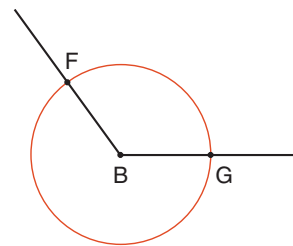
FB and BG are
2 sides of the
rhombus; FB = BG

Do not change the distance between
the compass and pencil points.
Place the compass point on F.
Draw a circle.
Place the compass point on G.
Draw a circle to intersect the second
circle you drew.
Label the point H where the circles intersect.

FH and HG are the
other 2 sides of
the rhombus.

Join FH and HG to form rhombus BFHG.
Draw a line through BH.
This line is the **angle bisector** of ∠FBG.
That is, ∠FBH = ∠HBG

BH is a diagonal
of the rhombus.

To check that the bisector of an angle has been
drawn correctly, we can:

➤ Measure the two angles formed by the bisector.
They should be equal.

➤ Fold the angle so the bisector is the fold line.
The two arms should coincide.

➤ Place a Mira along the angle bisector.
The reflection image of one arm of the angle
should coincide with the other arm, and vice versa.
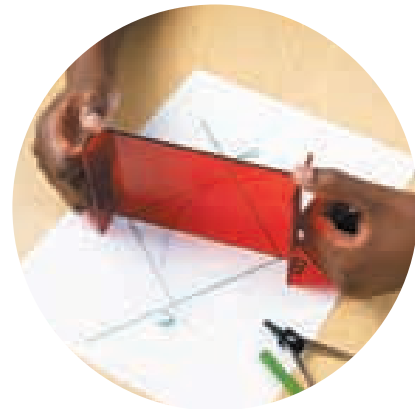
Practice

Show any construction lines.

1. Your teacher will give you a copy of this obtuse angle.
Use a Mira to bisect the angle.
Measure the two parts of the angle.
Are they equal?

A

B   C

2. Your teacher will give you a copy of this acute angle.
Use a plastic right triangle to bisect the angle.
Measure the two parts of the angle.
Are they equal?

D

E   F

3. Use a ruler and compass.
   a) Draw acute ∠PQR = 50°. Bisect the angle.
   b) Draw obtuse ∠GEF = 130°. Bisect the angle.

4. Draw a reflex angle of measure 270°.
   a) How many different methods can you find to bisect this angle?
   b) Describe each method.
   Check that the bisector you draw using each method is correct.
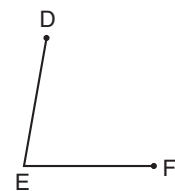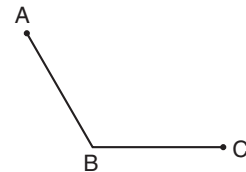
A *reflex angle* is an
angle between 180°
and 360°.

5. You have used Miras, triangles, and paper folding to bisect an angle. What is the advantage of using a ruler and compass?

6. a) Draw line segment HJ of length 8 cm.
Draw the perpendicular bisector of HJ.
b) Bisect each right angle in part a.
c) How many angle bisectors did you need to draw in part b? Explain why you needed this many bisectors.

7. **Assessment Focus** Your teacher will give you a large copy of this isosceles triangle. Use a ruler and compass.
a) Bisect ∠R.
b) Show that the bisector in part a is the perpendicular bisector of ST.
c) Is the result in part b true for:
  i) a different isosceles triangle?
  ii) an equilateral triangle?
  iii) a scalene triangle?
How could you find out? Show your work.
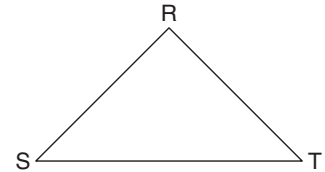
8. Describe examples of angle bisectors that you see in the environment.

9. **Take It Further** Your teacher will give you a copy of this triangle.
Cut it out.
Fold the triangle so BC and BA coincide. Open the triangle.
Fold it so AB and AC coincide. Open the triangle.
Fold it so AC and BC coincide. Open the triangle.
a) Measure the angles each crease makes at each vertex. What do you notice?
b) Label point K where the creases meet. Draw a circle in the triangle that touches each side of △ABC. What do you notice?
c) What have you constructed by folding?
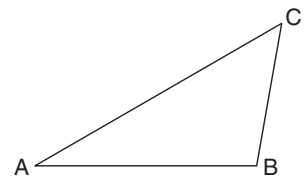
Reflect

How many bisectors can an angle have?
Draw a diagram to illustrate your answer.