3.2 Using Models to Multiply Fractions

Focus Use models to multiply fractions.

Describe a picture that shows 38×25.

Investigate

Work with a partner.
Model this problem with concrete materials.

One-quarter of a cherry pie was left over after dinner.
Graham ate one-half of the leftover pie for lunch the next day.
What fraction of the whole pie did he have for lunch?
What if Graham had eaten only one-quarter of the leftover pie.
What fraction of the whole pie would he have eaten?

Reflect & Share

How did you solve the problem?
Compare your solutions and strategies with those of another pair of classmates.
Was one strategy more efficient than another? Explain.

Connect

We can use different models to find the product of two fractions.
The following *Examples* show how we can use Pattern Blocks, counters, and a rectangle model.

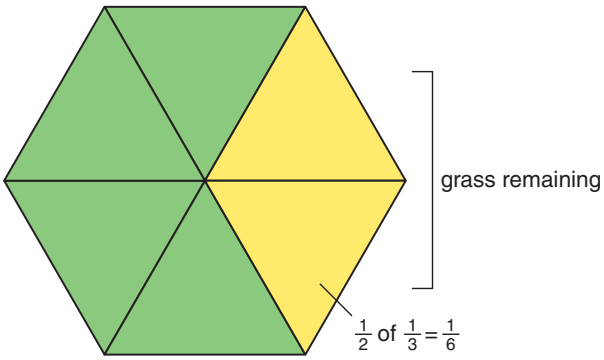
Example 1

Sandi cut $\frac{2}{3}$ of the grass on a lawn.

Akiva cut $\frac{1}{2}$ of the remaining grass.
What fraction of the lawn did Akiva cut?

A Solution

Use Pattern Blocks.
Let the yellow hexagon represent the lawn.
6 green triangles cover the yellow hexagon.
So, 6 green triangles also represent the lawn.
4 green triangles represent the grass cut by Sandi.
2 green triangles represent the remaining grass.



Akiva cut $\frac{1}{2}$ of the remaining grass.
One-half of the remaining grass is 1 green triangle.
One green triangle represents $\frac{1}{6}$.
So, $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$
Akiva cut $\frac{1}{6}$ of the lawn.

Example 2

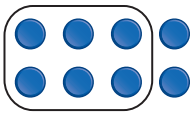
Multiply: $\frac{2}{3} \times \frac{6}{8}$

A Solution

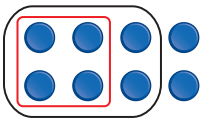
Use counters.

Think: We want $\frac{2}{3}$ of $\frac{6}{8}$ of one whole set of counters.

Model one whole set of eighths with eight counters.
There are 6 counters in $\frac{6}{8}$.
To model thirds, arrange the 6 counters into 3 equal groups.
Each group of 2 counters represents $\frac{1}{3}$.



So, $\frac{2}{3}$ of 6 counters is 4 counters.
But, 4 counters are part of a whole set of 8 counters.
So, 4 counters represent $\frac{4}{8}$, or $\frac{1}{2}$ of the original whole set.



So, $\frac{2}{3} \times \frac{6}{8} = \frac{1}{2}$

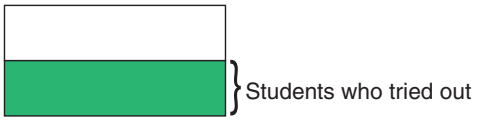
Example 3

One-half of the Grade 8 students tried out for the school's lacrosse team. Three-quarters of these students were successful.
What fraction of the Grade 8 students are on the team?

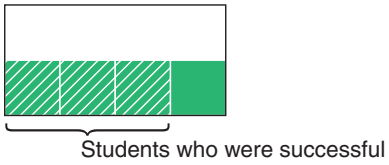
A Solution

Three-quarters of one-half of the Grade 8 students are on the team.

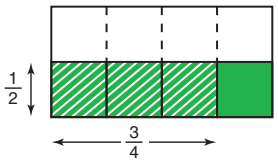
Draw a rectangle.
Show $\frac{1}{2}$ of the rectangle.



Divide $\frac{1}{2}$ of the rectangle into quarters.
Shade $\frac{3}{4}$.



Use broken lines to divide the whole rectangle into equal parts.
There are 8 equal parts.
Three parts are shaded.



$\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$

So, $\frac{3}{8}$ of the Grade 8 students are on the team.

Discuss the ideas

1. In *Example 1*, suppose Akiva cut $\frac{1}{4}$ of the remaining grass. Could we use Pattern Blocks to model the problem? Justify your answer.
2. In *Example 2*, why did we start with 8 counters?
   Could we have started with a different number of counters?
   Justify your answer.
3. Look at the area model in *Example 3*. The factors are fractions.
   How do the numbers of rows and columns in the product relate to the factors?
   How does the number of parts in the whole relate to the factors?
4. In *Example 3*, could you have divided the rectangle in half a different way? Would you get the same answer? Explain.

Practice

Check

5. Draw a rectangle to multiply $\frac{3}{5} \times \frac{1}{4}$.

a) Divide the rectangle into fourths vertically. How many fourths will you shade?

b) Divide the rectangle into fifths horizontally. How many fifths will you shade?

c) How many equal parts does the rectangle have?

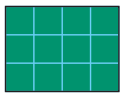
d) How many of these parts have been shaded twice?

e) What is the product of $\frac{3}{5} \times \frac{1}{4}$?

6. Copy each rectangle onto grid paper. Shade the rectangle to find each product.

a) $\frac{1}{2} \times \frac{3}{4}$

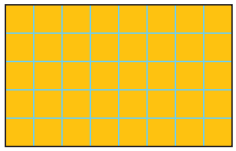
b) $\frac{3}{4} \times \frac{2}{3}$



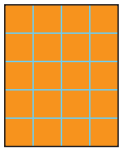
c) $\frac{2}{5} \times \frac{1}{2}$

d) $\frac{5}{6} \times \frac{1}{2}$

e) $\frac{3}{5} \times \frac{7}{8}$



f) $\frac{4}{5} \times \frac{3}{4}$



7. Use counters to find each product. Draw a diagram to record your work.

a) $\frac{3}{4} \times \frac{12}{15}$ **b)** $\frac{4}{5} \times \frac{10}{18}$ **c)** $\frac{1}{2} \times \frac{4}{12}$

d) $\frac{1}{4} \times \frac{8}{9}$ **e)** $\frac{5}{9} \times \frac{18}{24}$ **f)** $\frac{2}{3} \times \frac{15}{20}$

Apply

8. Find each product.

a) $\frac{3}{4} \times \frac{5}{8}$ **b)** $\frac{4}{9} \times \frac{2}{5}$

c) $\frac{1}{4} \times \frac{2}{3}$ **d)** $\frac{6}{7} \times \frac{2}{3}$

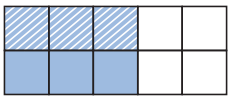
e) $\frac{2}{3} \times \frac{1}{3}$ **f)** $\frac{4}{5} \times \frac{4}{5}$

9. Write 3 multiplication statements using proper fractions. Make sure each statement is different from any statements you have worked with so far. Use a model to illustrate each product.

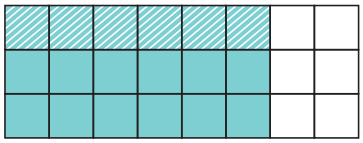
Recall that, in a proper fraction, the numerator is less than the denominator.

10. Write the multiplication equation represented by each diagram below.

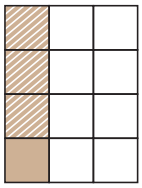
a)



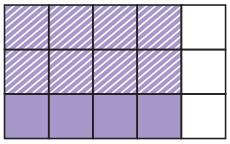
b)



c)



d)



11. Barry used $\frac{5}{8}$ of the money he had saved to buy a DVD player and 4 DVDs. The cost of the DVD player was $\frac{2}{5}$ of the amount he spent. What fraction of his savings did Barry spend on the DVD player?

12. **Assessment Focus**

a) Find each product.

i) $\frac{3}{4} \times \frac{2}{5}$ ii) $\frac{2}{4} \times \frac{3}{5}$

iii) $\frac{1}{4} \times \frac{3}{8}$ iv) $\frac{3}{4} \times \frac{1}{8}$

v) $\frac{3}{5} \times \frac{4}{6}$ vi) $\frac{3}{6} \times \frac{4}{5}$

b) What patterns do you see in the answers for part a? Use a model to illustrate the patterns.

c) Write some other products similar to those in part a.

Show your work.

13. Use a model to answer each question.

a) One-third of the students in a class wear glasses. One-half of the students who wear glasses are girls. What fraction of the class is girls who wear glasses?

b) John has $\frac{2}{3}$ of a tank of gas. He uses $\frac{3}{4}$ of the gas to get home. What fraction of a tank of gas does John use to get home? What fraction of the tank of gas is left?

c) Justin ate $\frac{3}{5}$ of a box of raisins. His sister then ate $\frac{1}{4}$ of the raisins left in the box. What fraction of the box of raisins did Justin's sister eat? What fraction of the box of raisins remained?

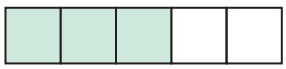
14. Gwen used the expression $\frac{4}{9} \times \frac{1}{5}$ to solve a word problem. What might the word problem be? Solve the problem.

15. **Take It Further** One-eighth of the seats in the movie theatre were empty. $\frac{2}{7}$ of the seats that were full were filled with teenagers. What fraction of the theatre was filled with teenagers?

16. **Take It Further** Why is $\frac{5}{8}$ of $\frac{3}{12}$ equal to $\frac{3}{8}$ of $\frac{5}{12}$? Use a model to explain your answer.

17. **Take It Further**

a) Look at the diagram.
How does it show $\frac{3}{5}$ of one whole?



b) Use the same diagram.
Explain how it shows that one whole is $\frac{5}{3}$ of $\frac{3}{5}$.

Reflect

When you use an area model to multiply two fractions, how do you decide how to draw the rectangle? Include an example in your explanation.