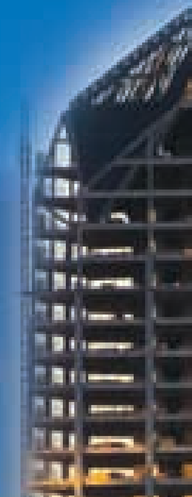
UNIT 1

Square Roots and the Pythagorean Theorem



Some of the greatest builders are also great mathematicians. They use concepts of geometry, measurement, and patterning.

Look at the architecture on these pages. What aspects of mathematics do you see?

In this unit, you will develop strategies to describe distances that cannot be measured directly.

What You'll Learn

- Determine the square of a number.
- Determine the square root of a perfect square.
- Determine the approximate square root of a non-perfect square.
- Develop and apply the Pythagorean Theorem.

Why It's Important

The Pythagorean Theorem enables us to describe lengths that would be difficult to measure using a ruler. It enables a construction worker to make a square corner without using a protractor.

Key Words

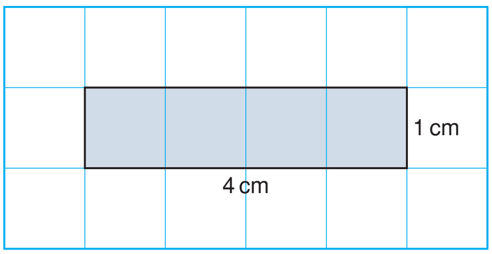
- square number
- perfect square
- square root
- leg
- hypotenuse
- Pythagorean Theorem
- Pythagorean triple

1.1 Square Numbers and Area Models

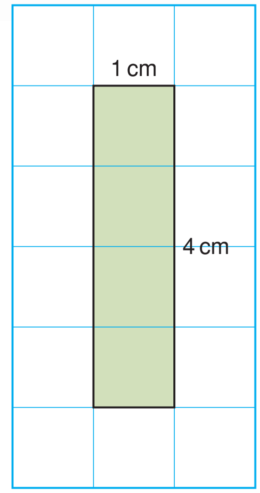
Focus Relate the area of a square and square numbers.

A rectangle is a quadrilateral with 4 right angles.
A square also has 4 right angles.

A rectangle with base 4 cm and height 1 cm
is the same as a rectangle with base 1 cm and height 4 cm.

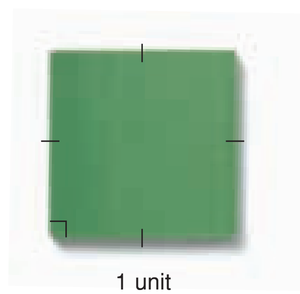


These two rectangles are *congruent*.
Is every square a rectangle?
Is every rectangle a square?



Investigate

Work with a partner.
You will need grid paper and 20 square tiles like this:
Use the tiles to make as many different rectangles
as you can with each area.



4 square units	12 square units
6 square units	16 square units
8 square units	20 square units
9 square units	

Draw the rectangles on grid paper.

➤ For how many areas above were you able to make a square?
➤ What is the side length of each square you made?
➤ How is the side length of a square related to its area?

Reflect & Share

Compare your strategies and results with those of another pair of classmates.
Find two areas greater than 20 square units for which you could use tiles to make a square.
How do you know you could make a square for each of these areas?

Connect

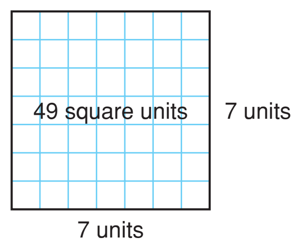
When we multiply a number by itself, we *square* the number.
For example: The square of 4 is $4 \times 4 = 16$.
We write: $4 \times 4 = 4^2$
So, $4^2 = 4 \times 4 = 16$
We say: Four squared is sixteen.
16 is a **square number**, or a **perfect square**.

One way to model a square number is to draw a square whose area is equal to the square number.

Example 1

Show that 49 is a square number.
Use a diagram, symbols, and words.

A Solution

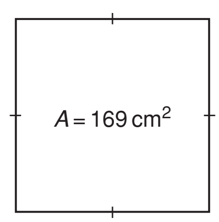


Draw a square with area 49 square units.
The side length of the square is 7 units.
Then, $49 = 7 \times 7 = 7^2$
We say: Forty-nine is seven squared.

Example 2

A square picture has area 169 cm^2.
Find the perimeter of the picture.

A Solution



The picture is a square with area 169 cm^2.
Find the side length of the square:

Find a number which, when multiplied by itself, gives 169.
$13 \times 13 = 169$
So, the picture has side length 13 cm.

Perimeter is the distance around the picture.
So, $P = 13\text{ cm} + 13\text{ cm} + 13\text{ cm} + 13\text{ cm}$
$= 52\text{ cm}$
The perimeter of the picture is 52 cm.

Discuss the ideas

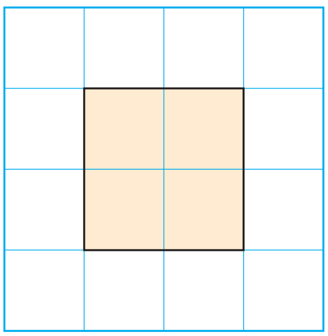
1. Is 1 a square number? How can you tell?
2. Suppose you know the area of a square. How can you find its perimeter?
3. Suppose you know the perimeter of a square. How can you find its area?

Practice

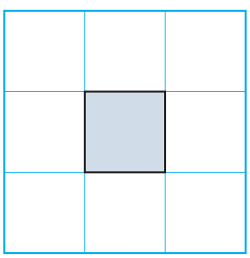
Check

4. Match each square below to its area.

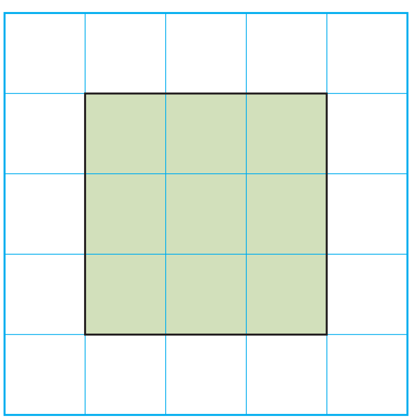
a)



b)



c)



i) 1 unit × 1 unit = 1 square unit
ii) 2 units × 2 units = 4 square units
iii) 3 units × 3 units = 9 square units

5. Find the area of a square with each side length.
   a) 8 units b) 10 units c) 3 units

6. Use square tiles.
   Make as many different rectangles as you can with area 36 square units.
   Draw your rectangles on grid paper.
   Is 36 a perfect square?
   Justify your answer.

Apply

7. Use square tiles.
   Make as many different rectangles as you can with area 28 square units.
   Draw your rectangles on grid paper.
   Is 28 a perfect square?
   Justify your answer.

8. Show that 25 is a square number.
   Use a diagram, symbols, and words.

9. Show that 12 is not a square number.
   Use a diagram, symbols, and words.

10. Use a diagram to show that each number below is a square number.

a) 1 **b)** 144
c) 121 **d)** 900

11. Find the side length of a square with each area.

a) 100 m^2 **b)** 64 cm^2
c) 81 m^2 **d)** 400 cm^2

12. Which of these numbers is a perfect square?
How do you know?

a) 10 **b)** 50
c) 81 **d)** 20

13. Use 1-cm grid paper.
Draw as many different rectangles as you can with area 64 cm^2.
Find the base and height of each rectangle.
Record the results in a table.

Base (cm)	Height (cm)	Perimeter (cm)

Which rectangle has the least perimeter?
What can you say about this rectangle?

14. I am a square number.
The sum of my digits is 9.
What square numbers might I be?

15. These numbers are not square numbers.
Which two consecutive square numbers is each number between?
Describe the strategy you used.

a) 12 **b)** 40
c) 75 **d)** 200

16. The floor of a large square room has area 144 m^2.

a) Find the length of a side of the room.
b) How much baseboard is needed to go around the room?
c) Each piece of baseboard is 2.5 m long.
How many pieces of baseboard are needed?
What assumptions do you make?

17. A garden has area 400 m^2.
The garden is divided into 16 congruent square plots.
Sketch a diagram of the garden.
What is the side length of each plot?

18. **Assessment Focus** Which whole numbers between 50 and 200 are perfect squares?
Explain how you know.

19. Lee is planning to fence a square kennel for her dog.
Its area must be less than 60 m^2.

a) Sketch a diagram of the kennel.
b) What is the kennel's greatest possible area?
c) Find the side length of the kennel.
d) How much fencing is needed?
e) One metre of fencing costs $10.00.
What is the cost of the fencing?
What assumptions do you make?

20. **Take It Further** Devon has a piece of poster board 45 cm by 20 cm.
His teacher challenges him to cut the board into parts, then rearrange the parts to form a square.

a) What is the side length of the square?
b) What are the fewest cuts Devon could have made?
Explain.

21. **Take It Further** The digital root of a number is the result of adding the digits of the number until a single-digit number is reached.
For example, to find the digital root of 147:
$1 + 4 + 7 = 12$ and $1 + 2 = 3$

a) Find the digital roots of the first 15 square numbers.
What do you notice?
b) What can you say about the digital root of a square number?
c) Use your results in part b.
Which of these numbers might be square numbers?
i) 440 **ii)** 2809 **iii)** 3008
iv) 4225 **v)** 625

Reflect

Use diagrams to explain why 24 is not a square number but 25 is a square number.

1.2 Squares and Square Roots

Focus Find the squares and square roots of whole numbers.

A factor is a number that divides exactly into another number.
For example, 1, 2, 3, and 6 are factors of 6.
What are the factors of 10?

Investigate

Work with a partner.
Your teacher will give you a copy of this chart.
It shows the factors of each whole number from 1 to 8.
Complete your copy of the chart.

					6		8																						
			4		3		4																						
	2	3	2	5	2	7	2																						
1	1	1	1	1	1	1	1																						
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**

➤ Which numbers have only two factors?
What do you notice about these numbers?

➤ Which numbers have an even number of factors, but more than 2 factors?

➤ Which numbers have an odd number of factors?

Reflect & Share

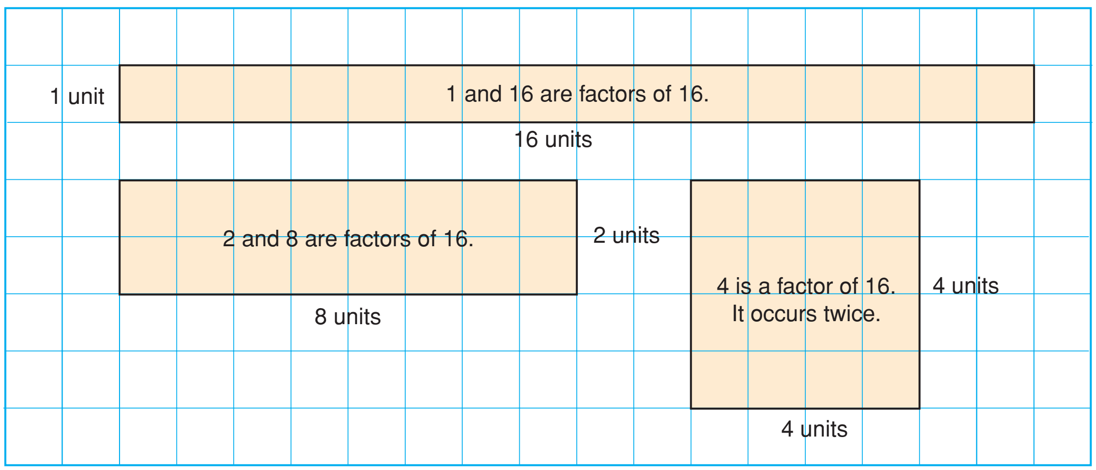
Compare your chart and results with those of another pair of classmates.
Which numbers in the chart are square numbers? How do you know?
What seems to be true about the factors of a square number?

Connect

➤ Here are some ways to tell whether a number is a square number.
If we can find a division sentence for a number so that the quotient is equal to the divisor, the number is a square number.
For example, $16 \div 4 = 4$, so 16 is a square number.

dividend **divisor** **quotient**

➤ We can also use factoring.
Factors of a number occur in pairs.
These are the dimensions of a rectangle.



Sixteen has 5 factors: 1, 2, 4, 8, 16

A factor that occurs twice is only written once in the list of factors.

Since there is an odd number of factors, one rectangle is a square.
The square has side length 4 units.
We say that 4 is a **square root** of 16.
We write: $4 = \sqrt{16}$

When a number has an odd number of factors, it is a square number.

When we multiply a number by itself, we square the number.
Squaring and taking the square root are inverse operations. That is, they undo each other.

$4 \times 4 = 16$ $\qquad$ $\sqrt{16} = \sqrt{4 \times 4} = \sqrt{4^2}$

so, $4^2 = 16$ $\qquad$ $= 4$

Example 1

Find the square of each number.

a) 5 **b)** 15

A Solution

a) The square of 5 is: $5^2 = 5 \times 5$
$= 25$

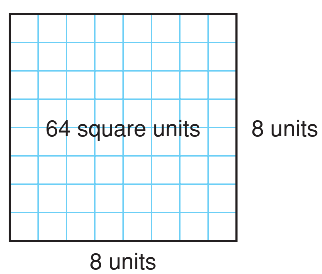
b) The square of 15 is: $15^2 = 15 \times 15$
$= 225$

Example 2

Find a square root of 64.

A Solution

Use grid paper.
Draw a square with area 64 square units.
The side length of the square is 8 units.
So, $\sqrt{64} = 8$



Example 2
Another Solution

Find pairs of factors of 64.
Use division facts.

$64 \div 1 = 64$	1 and 64 are factors.
$64 \div 2 = 32$	2 and 32 are factors.
$64 \div 4 = 16$	4 and 16 are factors.
$64 \div 8 = 8$	8 is a factor. It occurs twice.

The factors of 64 are: 1, 2, 4, **8**, 16, 32, 64
A square root of 64 is 8, the factor that occurs twice.

Example 3

The factors of 136 are listed in ascending order.
136: 1, 2, 4, 8, 17, 34, 68, 136
Is 136 a square number?
How do you know?

Numbers are in ascending order when they are written from least to greatest number.

A Solution

A square number has an odd number of factors.
One hundred thirty-six has 8 factors.
Eight is an even number.
So, 136 is not a square number.

Example 3
Another Solution

List the factors of 136 in a column, in ascending order.
Beside this column, list the factors in descending order.
Multiply the numbers in each row.

$$
\begin{aligned}
1 \times 136 &= 136\\
2 \times 68 &= 136\\
4 \times 34 &= 136\\
8 \times 17 &= 136\\
17 \times 8 &= 136\\
34 \times 4 &= 136\\
68 \times 2 &= 136\\
136 \times 1 &= 136
\end{aligned}
$$

The same factor does not occur in the same place in both columns.
So, 136 cannot be written as the product of 2 equal numbers.

So, 136 is not a square number.

Discuss the ideas

1. Squaring a number and taking a square root are inverse operations. What other inverse operations do you know?
2. When the factors of a perfect square are written in order from least to greatest, what do you notice?
3. Why do you think numbers such as 4, 9, 16, … are called perfect squares?
4. Suppose you list the factors of a perfect square. Why is one factor a square root and not the other factors?

Practice

Check

5. Find the square of each number.

a) 4 **b)** 6
c) 2 **d)** 9

6. Find.

a) 8^2 **b)** 3^2
c) 1^2 **d)** 7^2

7. Find a square root of each number.

a) 25 **b)** 81
c) 64 **d)** 169

8. **a)** Find the square of each number.

i) 1 **ii)** 10
iii) 100 **iv)** 1000

b) Use the patterns in part a. Predict the square of each number.

i) 10 000 **ii)** 1 000 000

9. **a)** Use a table like this.

Number = 50	
Factor Pairs	
1	50
2	

List the factor pairs of each number. Which numbers are square numbers? How do you know?

i) 50 **ii)** 100
iii) 144 **iv)** 85

b) Find a square root of each square number in part a.

Apply

10. List the factors of each number in ascending order.
Find a square root of each number.

a) 256 **b)** 625 **c)** 121

11. The factors of each number are listed in ascending order.
Which numbers are square numbers?
How do you know?

a) 225: 1, 3, 5, 9, 15, 25, 45, 75, 225
b) 500: 1, 2, 4, 5, 10, 20, 25, 50, 100, 125, 250, 500
c) 324: 1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 81, 108, 162, 324
d) 160: 1, 2, 4, 5, 8, 10, 16, 20, 32, 40, 80, 160

12. **a)** List the factors of each number in ascending order.

i) 96 **ii)** 484
iii) 240 **iv)** 152
v) 441 **vi)** 54

b) Which numbers in part a are square numbers?
How can you tell?

13. Find each square root.

a) $\sqrt{1}$ **b)** $\sqrt{49}$
c) $\sqrt{144}$ **d)** $\sqrt{9}$
e) $\sqrt{16}$ **f)** $\sqrt{100}$
g) $\sqrt{625}$ **h)** $\sqrt{225}$

14. Find a square root of each number.

a) 3^2 b) 6^2

c) 10^2 d) 117^2

15. Find the square of each number.

a) $\sqrt{4}$ b) $\sqrt{121}$

c) $\sqrt{225}$ d) $\sqrt{676}$

16. **Assessment Focus** Find each square root. Use a table, list, or diagram to support your answer.

a) $\sqrt{169}$

b) $\sqrt{36}$

c) $\sqrt{196}$

17. Find the number whose square root is 23. Explain your strategy.

18. Use your results from questions 6b and 13d. Explain why squaring and taking the square root are inverse operations.

19. Order from least to greatest.

a) $\sqrt{36}$, 36, 4, $\sqrt{9}$

b) $\sqrt{400}$, $\sqrt{100}$, 19, 15

c) $\sqrt{81}$, 81, $\sqrt{100}$, 11

d) $\sqrt{49}$, $\sqrt{64}$, $\sqrt{36}$, 9

20. Which perfect squares have square roots between 1 and 20?
How do you know?

21. **Take It Further**

a) Find the square root of each palindromic number.

> A palindromic number is a number that reads the same forward and backward.

i) $\sqrt{121}$

ii) $\sqrt{12\,321}$

iii) $\sqrt{1\,234\,321}$

iv) $\sqrt{123\,454\,321}$

b) Continue the pattern.
Write the next 4 palindromic numbers in the pattern and their square roots.

22. **Take It Further**

a) Find.

i) 2^2 ii) 3^2

iii) 4^2 iv) 5^2

b) Use the results from part a.
Find each sum.

i) $2^2 + 3^2$ ii) $3^2 + 4^2$

iii) $2^2 + 4^2$ iv) $3^2 + 5^2$

c) Which sums in part b are square numbers?
What can you say about the sum of two square numbers?

Reflect

Which method of determining square numbers are you most comfortable with?
Justify your choice.