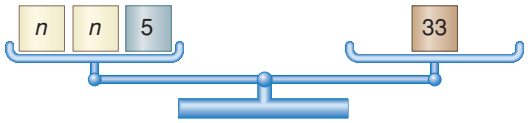
6.2 Solving Equations Using Algebra

Focus Solve a linear equation symbolically.

Investigate

Work with a partner to solve this problem.

Asuka invites some friends over to celebrate her spelling bee award.
Her mom has some veggie burgers, but she doesn't have enough.
Asuka's mom tells her that she needs twice as many veggie burgers, plus 5 more.
Her mom leaves to run errands, and Asuka has forgotten to ask how many veggie burgers her mom has.
There will be 33 people at the party—Asuka and 32 friends.
Asuka uses this diagram to model the situation:



What does n represent?
Write an equation for the balance-scales model.
How many different ways can you solve the equation?
Show each way.

Reflect & Share

Compare the equation you wrote with that of another pair of classmates.
If the equations are different, is each equation correct?
How can you find out?
Discuss the strategies you used to solve the equation.
If you did not solve the equation using algebra, work together to do that now.

Connect

We can solve an equation using algebra.
To help us visualize the equation, we think about a model,
such as balance scales or algebra tiles.

To solve an equation, we need to isolate the variable on one side of the equation.
To do this, we get rid of the numbers on that side of the equation.

When we solve an equation using algebra, we must also preserve the equality.
Whatever we do to one side of the equation, we must do to the other side, too.

Example 1

Fabian charges $3 for each bag of leaves he rakes, and $5 for mowing the lawn.
On Sunday, Fabian mowed 1 lawn and raked leaves. He earned $14.
How many bags of leaves did Fabian rake?

a) Write an equation to represent this problem.

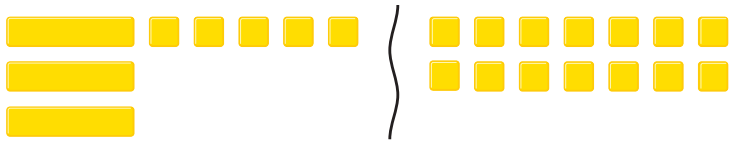
b) Use algebra tiles to solve the equation.
Use symbols to record each step.

c) Verify the solution using algebra.

A Solution

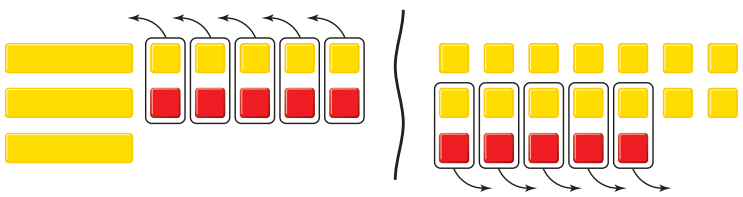
a) Let b represent the number of bags of leaves Fabian raked.
An equation is: $3b + 5 = 14$

b) Model the equation with tiles.



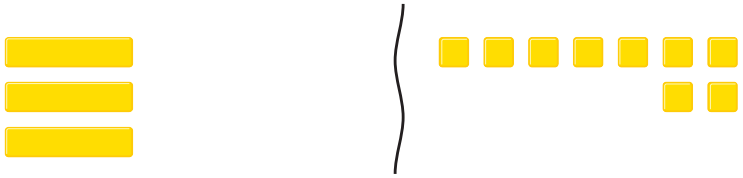
$3b + 5 = 14$

Add 5 negative unit tiles to each side to isolate the variable.
Remove zero pairs.



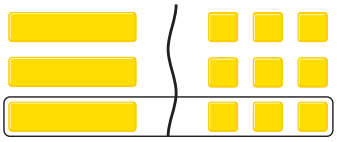
$3b + 5 - 5 = 14 - 5$

These tiles remain.



$3b = 9$

Divide each side into 3 equal groups.



$\frac{3b}{3} = \frac{9}{3}$

$b = 3$

Fabian raked 3 bags of leaves.

c) To verify the solution, substitute $b = 3$ into $3b + 5 = 14$.

Left side $= 3b + 5$ Right side $= 14$

$= 3(3) + 5$

$= 9 + 5$

$= 14$

Since the left side equals the right side, $b = 3$ is correct.

Fabian raked 3 bags of leaves.

It can be inconvenient to model an equation using algebra tiles when large numbers are involved, or when the solution is a fraction or a decimal.

Example 2

a) Use algebra to solve the equation:

$16t - 69 = -13$

b) Verify the solution.

A Solution

a) $16t - 69 = -13$

To isolate the variable term, add 69 to each side.

$16t - 69 + 69 = -13 + 69$

$16t = 56$

To isolate the variable, divide each side by 16.

$\frac{16t}{16} = \frac{56}{16}$

At this stage in the solution, we can continue in 2 ways.

As a fraction:

$t = \frac{56}{16}$

Divide by the common factor 8.

$t = \frac{56 \div 8}{16 \div 8}$

$t = \frac{7}{2}$

As a decimal:

$t = \frac{56}{16}$

Use a calculator.

$t = 3.5$

b) To verify the solution, substitute $t = \frac{7}{2}$ into:

$16t - 69 = -13$

Left side $= 16t - 69$

$= 16\left(\frac{7}{2}\right) - 69$

$= {}^{8}\cancel{16}\left(\frac{7}{\cancel{2}}\right)_{1} - 69$

$= 56 - 69$

$= -13$

Right side $= -13$

Since the left side equals the right side, $t = \frac{7}{2}$ is correct.

To verify the solution, substitute $t = 3.5$ into:

$16t - 69 = -13$

Left side $= 16t - 69$

$= 16(3.5) - 69$

$= 56 - 69$

$= -13$

Right side $= -13$

Since the left side equals the right side, $t = 3.5$ is correct.

Discuss the ideas

1. When is it easier to solve an equation using algebra instead of using algebra tiles or a balance-scales model?
2. When is it easier to verify a solution using substitution instead of algebra tiles or a balance-scales model?
3. When you solve an equation using algebra, why do you add or subtract the same number on each side? Why do you divide each side by the coefficient of the variable term?
4. Suppose the solution to an equation is a fraction or a decimal. Which would you prefer to use to verify the solution? Give reasons for your answer.

Practice

Check

5. Model each equation. Then solve it using concrete materials.
Use algebra to record each step you take. Verify each solution.

a) $2x - 1 = 7$
b) $11 = 4a - 1$
c) $5 + 2m = 9$
d) $1 = 10 - 3x$
e) $13 - 2x = 5$
f) $3x - 6 = 12$

6. Use algebra to solve each equation. Verify the solution.

a) $4x = -16$
b) $12 = -3x$
c) $-21 = 7x$
d) $6x = -30$

7. Check each student's work. Rewrite a correct and complete algebraic solution where necessary.

a) Student A:

$$-3x + 15 = 30$$
$$-3x + 15 - 15 = 30 + 15 - 15$$
$$-3x = 30$$
$$\frac{-3x}{-3} = \frac{30}{-3}$$
$$x = -10$$

b) Student B:

$$7 = 1 + 2n$$
$$7 - 1 = 1 - 1 + 2n$$
$$8 = 2n$$
$$\frac{8}{2} = \frac{2n}{2}$$
$$4 = n$$
$$n = 4$$

c) Student C:

$$3 + 2t = 4$$
$$3 + 2t - 3 = 4 - 3$$
$$2t = 1$$
$$t = 2$$

d) Student D:

$$-5 = -8 + 5f$$
$$-5 + 8 = -8 + 8 + 5f$$
$$3 = 5f$$
$$\frac{3}{5} = \frac{5f}{5}$$
$$\frac{3}{5} = f$$
$$f = \frac{3}{5}$$

Apply

8. Solve each equation.
Verify the solution.

a) $2x + 5 = -7$ **b)** $-3x + 11 = 2$
c) $-9 = 5 + 7x$ **d)** $18 = -4x + 2$

9. Navid now has \$72 in her savings account. Each week she will save \$24. After how many weeks will Navid have a total savings of \$288?

a) Write an equation you can use to solve the problem.
b) Solve the equation.
When will Navid have \$288 in her savings account?
c) Verify the solution.

10. **Assessment Focus**

The Grade 8 students had an end-of-the-year dance.
The disc jockey charged $85 for setting up the equipment, plus $2 for each student who attended the dance.
The disc jockey was paid $197.
How many students attended the dance?

a) Write an equation you can use to solve the problem.
b) Solve the equation.
c) Check your answer and explain how you know it is correct.

11. Solve each equation.
Verify your solution.

a) $-8x + 11 = 59$ **b)** $11c + 21 = -34$
c) $23 = -5b + 3$ **d)** $-45 = 6a - 15$
e) $52 = 25 - 9f$ **f)** $-13 + 4d = 31$

12. Solve each equation.
Verify your solution.

a) $3n + 7 = 8$ **b)** $6x + 6 = 15$
c) $-23 = 5p - 27$ **d)** $5p + 6 = 7$
e) $8e - 9 = -3$ **f)** $-17 + 10g = -9$

13. The high temperature today is 7°C higher than twice the high temperature yesterday.
The high temperature today is –3°C. What was the high temperature yesterday?

a) Write an equation you can use to solve the problem.
b) Solve the equation.
Verify the solution.

14. **Take It Further**

Use this information.
Boat rental: $300
Fishing rod rental: $20

a) Write a problem that can be solved using an equation.
b) Write the equation, then solve the problem.
c) How could you have solved the problem without writing an equation? Explain.

15. **Take It Further**

Use this information:
Water is pumped out of a flooded basement at a rate of 15 L/min.

a) Write a problem that can be solved using an equation.
b) Write, then solve, the equation.

Reflect

Which types of equations do you prefer to solve using algebra?
Explain why you may not want to use algebra tiles or a balance-scales model.