1.5 Number Patterns

Focus Describe, extend, and explain patterns.

Explore

Work on your own.
Blaise Pascal lived in France in the 17th century.
He was 13 years old when he constructed the triangle below.
This triangle is called Pascal's Triangle.

1	row 1
1 1	row 2
1 2 1	row 3
1 3 3 1	row 4
1 4 6 4 1	row 5
1 5 10 10 5 1	row 6

➢ What patterns do you see in the triangle?
➢ What symmetry do you see in the triangle?

Reflect & Share

Compare your patterns with those of a classmate.
Together, write about three different patterns you see in the triangle.

Connect

Here are some of the patterns in Pascal's Triangle.

➢ Each row begins and ends with 1.
After the second row, each number is the sum of the 2 numbers above it.
To write row 7:
Start with 1.
Add: 1 + 5 = 6
Add: 5 + 10 = 15
Add: 10 + 10 = 20, and so on

	Sum
1	1
1 1	2
1 2 1	4
1 3 3 1	8
1 4 6 4 1	16
1 5 10 10 5 1	32
1 6 15 20 15 6 1	64

➢ The sum of the numbers in each row is shown above, and in the table on the next page.
From the 2nd row on, the sums can be written as powers.

Row	2	3	4	5	6	7
Sum in standard form	2	4	8	16	32	64
Sum in exponent from	2^1	2^2	2^3	2^4	2^5	2^6

We can use this table to predict the sum of the numbers in any row. All sums are powers of 2.
The exponent is 1 less than the row number.
So, the 10th row has sum: $2^9 = 512$
And the 19th row has sum: $2^{18} = 262\ 144$

➢ The 3rd numbers in each row have this pattern: 1, 3, 6, 10, 15, ...
To get each term in the pattern, we add 1 more than we added before. We can use this to extend the pattern.
The 5th term: 15
The 6th term: $15 + 6 = 21$
The 7th term: $21 + 7 = 28$, and so on

Example

Describe each pattern in words. Write the next 3 terms.

a) 4, 9, 14, 19, … **b)** 1, 3, 9, 27, … **c)** 1, 3, 7, 13, 21, …

Solution

a) 4, 9, 14, 19, …
Start at 4.
Add 5 to get the next number.
The next 3 terms are 24, 29, 34.

b) 1, 3, 9, 27, …
Start at 1.
Multiply by 3 to get the next number.
The next 3 terms are 81, 243, 729.

c) 1, 3, 7, 13, 21, …
Start at 1.
Add 2.
Increase the number added by 2 each time.
The next 3 terms are 31, 43, 57.

Practice

1. Write the next 3 terms in each pattern.

a) 7, 9, 11, 13, …
b) 1, 5, 25, 125, …
c) 4, 7, 10, 13, …
d) 1, 10, 100, 1000, …
e) 20, 19, 18, 17, …
f) 79, 77, 75, 73, …

2. Write the next 3 terms in each pattern.

a) 3, 4, 6, 9, …
b) 1, 4, 9, 16, …
c) 101, 111, 121, 131, …
d) 1, 12, 123, 1234, …
e) 1, 4, 16, 64, …
f) 256, 128, 64, 32, …

3. Describe each pattern in words.
Write the next 3 terms.

a) 200, 199, 201, 198, …
b) 4, 7, 12, 19, …
c) 100, 99, 97, 94, …
d) 2, 6, 12, 20, …
e) 50, 48, 44, 38, …
f) 2, 6, 18, 54, …

4. Create your own number pattern. Trade patterns with a classmate. Describe your classmate's pattern.
Write the next 3 terms.

5. a) Copy this pattern. Find each product.

$99 \times 11 = \square$	$99 \times 111 = \square$	…
$99 \times 22 = \square$	$99 \times 222 = \square$	…
$99 \times 33 = \square$	$99 \times 333 = \square$	…
⋮	⋮	

b) Extend this pattern sideways and down.
Predict the next 6 terms in each row and column.
c) Check your predictions with a calculator.

6. This pattern shows the first 3 triangular numbers.

a) Draw the next 3 terms in the pattern.
b) List the first 6 triangular numbers.
c) Find the next 2 triangular numbers without drawing pictures.
Explain how you did this.

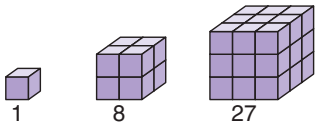
Number Strategies

One can of pop contains 355 mL. About how many 2-L pop bottles can be filled with one case of 24 cans of pop?

d) Add consecutive triangular numbers;
that is, Term 1 + Term 2; Term 2 + Term 3; and so on.
What pattern do you see?
Write the next 3 terms in this pattern.

e) Subtract consecutive triangular numbers;
that is, Term 2 − Term 1; Term 3 − Term 2; and so on.
What pattern do you see?
Write the next 3 terms in this pattern.

7. This pattern shows the first 3 cube numbers.



a) Sketch the next 3 cube numbers in the pattern.
Use interlocking cubes if they help.

b) Write the next 3 cube numbers without drawing pictures.
Explain how you did this.

8. **Assessment Focus**

a) Write the first 10 powers of 2; that is, 2^1 to 2^{10}, in standard form.

b) What pattern do you see in the units digits?

c) How can you use this pattern to find the units digit of 2^{40}?

d) Investigate powers of other numbers.
Look for patterns in the units digits.
Explain how you can use these patterns to find units digits for powers too large to display on the calculator.

Take It Further

9. Some sequences of numbers may represent different patterns.
Extend each pattern in as many different ways as you can.
Write the pattern rule for each pattern.

a) 1, 2, 4, … **b)** 1, 4, 9, … **c)** 5, 25, …

Reflect

Choose 3 different types of patterns from this section.
Describe each pattern.
Explain how you can use the pattern to predict the next term.

Using Different Strategies

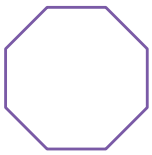
Problem

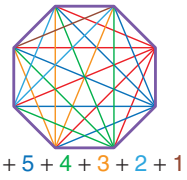
There are 8 people at a party.
Each person shakes hands with everyone else.
How many handshakes are there?

Think of a strategy

Strategy 1: **Draw a diagram and count.**

Draw 8 dots. Join every dot to every other dot.
Count the line segments.

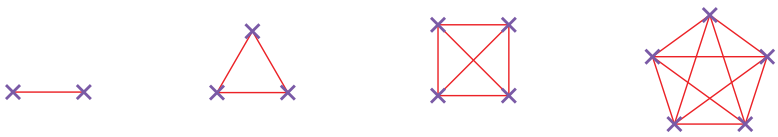




Number of line segments
$= 8 + 5 + 5 + 4 + 3 + 2 + 1$
$= 28$

There are 28 handshakes.

Strategy 2: **Solve simpler problems, then look for a pattern.**



2 people	3 people	4 people	5 people
1 handshake	3 handshakes	6 handshakes	10 handshakes

Make a table.
Each time you add a person,
you add one more
handshake than the time before.
So, 6 people: 10 + 5, or 15 handshakes
7 people: 15 + 6, or 21 handshakes
8 people: 21 + 7, or 28 handshakes

Number of People	Number of Handshakes	
2	1	
3	3	+2
4	6	+3
5	10	+4